Glazer & Wark

8.1 Show
$$\vec{E} = \begin{bmatrix} A_x \cos(\frac{n_x}{a}\pi x)\sin(\frac{n_y}{a}\pi y)\sin(\frac{n_z}{a}\pi z)e^{i\omega t} \\ A_y \sin(\frac{n_x}{a}\pi x)\cos(\frac{n_y}{a}\pi y)\sin(\frac{n_z}{a}\pi z)e^{i\omega t} \\ A_z \sin(\frac{n_x}{a}\pi x)\sin(\frac{n_y}{a}\pi y)\cos(\frac{n_z}{a}\pi z)e^{i\omega t} \end{bmatrix}.$$

is an wave equation for ~~light~~ Electric field inside box ~~with no charge~~ of width a.

Explicitly, $\vec{\nabla}^2 = \vec{\nabla}\cdot\vec{\nabla} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right).$

$$\Rightarrow \vec{\nabla}^2\vec{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{E}$$

$$= -(k_x^2 + k_y^2 + k_z^2)\vec{E} = -k^2\vec{E}$$

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\vec{E} = -\frac{\omega^2}{c^2}\vec{E}. \qquad \frac{\omega^2}{c^2} = k^2 \text{ by dispersion relation of light.}$$

It satisfies the boundary condition of ~~...~~ $\vec{E}$ inside conductor since the ~~per~~ tangent components of the ~~wave~~ ~~polarization~~ field is 0.

To check, let $x = a$, then $\vec{E} = \begin{bmatrix} A_x \\ 0 \\ 0 \end{bmatrix} e^{i\omega t}$.

Now for $\vec{\nabla}\cdot E$, notice
$$\vec{\nabla}\cdot\vec{E} = -k_x\tan(k_x x)E_x - k_y\tan(k_y y)E_y - k_z\tan(k_z z)E_z.$$

$$= -\sin(k_x)\sin(k_y)\sin(k_z)e^{i\omega t}\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}\cdot\begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix}$$

$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}\cdot\begin{pmatrix} k_x \\ k_y \\ k_z \end{pmatrix} = 0$ can be seen by partial condition of light.

$$\Rightarrow \vec{\nabla}\cdot\vec{E} = 0.$$