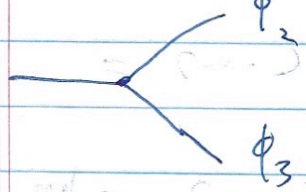
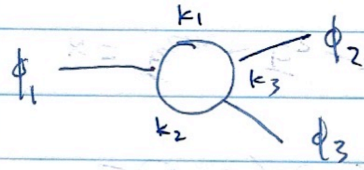
Schwartz

7.1 (a)



$ig/3$

(b)



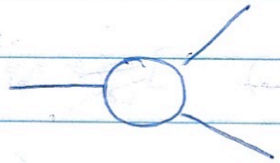
$$\left(\frac{ig}{3!}\right)^3 \frac{i}{k_1^2-m^2+i\varepsilon}\,\frac{i}{k_2^2-m^2+i\varepsilon}\,\frac{i}{k_3^2-m^2+i\varepsilon}$$

$$= \left(\frac{g}{6}\right)^3 (-1)\,\frac{1}{k_1^2-m^2+i\varepsilon}\,\frac{1}{k_2^2-m^2+i\varepsilon}\,\frac{1}{k_3^2-m^2+i\varepsilon}$$

Schwartz

7.1 (c) $\langle 0|T\{\phi_1\phi_2\phi_3 \exp[i\int d^4x\, \mathcal{L}_I[\phi_0(x)]]\}|0\rangle$

...



is in 3rd order,

$$T\{\exp[i\int d^4x\, \mathcal{L}_I[\phi_0(x)]]\}$$

$$= 1 + i\int d^4x\, \mathcal{L}_I[\phi_x] + \frac{i^2}{2}\int d^4x\, d^4x'\, T\{\mathcal{L}_I[\phi_x]\mathcal{L}_I[\phi_{x'}]\}$$

$$+ \frac{i^3}{3!}\int d^4x\, d^4x'\, d^4x''\, T\{\mathcal{L}_I[\phi_x]\mathcal{L}_I[\phi_{x'}]\mathcal{L}_I[\phi_{x''}]\}.$$

$O(g^3)$

$$+ \cdots$$

$$\langle 0|\frac{i^3}{3!}\int d^4x\, d^4x'\, d^4x''\, T\{\phi_1\phi_2\phi_3\, \mathcal{L}_I[\phi_x]\mathcal{L}_I[\phi_{x'}]\mathcal{L}_I[\phi_{x''}]\}|0\rangle$$

Letting $\mathcal{L}_I[\phi_0] = \frac{g}{3!}\phi_0^3$, we have

$$\frac{1}{3!}\left(\frac{ig}{3!}\right)^3\langle 0|\int d^4x\, d^4x'\, d^4x''\, T\{\phi_1\phi_2\phi_3\, \phi_x^3\, \phi_{x'}^3\, \phi_{x''}^3\, |0\rangle$$

$$= \frac{1}{3!}\left(\frac{ig}{3!}\right)^3 \int d^4x\, d^4x'\, d^4x'' \langle 0| T\{\phi_1\phi_2\phi_3\, \phi_x^3\, \phi_{x'}^3\, \phi_{x''}^3\}|0\rangle .$$

$$T\{\phi_1\phi_2\phi_3\, \phi_x^3\, \phi_{x'}^3\, \phi_{x''}^3\}$$

$$\left.\begin{aligned}
&= D_{1x}\, D_{2x'}\, D_{3x''}\, D_{xx'}\, D_{x'x''}\, D_{xx''} && \times 6 \\
&+ D_{1x'}\, D_{2x''}\, D_{3x}\, (\,-\,-\,-\,) && \times 6 \\
&+ \quad \vdots && \vdots \\
&\ D_{1x}\, D_{2x''}\, D_{3x'}\, (\,-\,-\,-\,-\,) && \times 6
\end{aligned}\right\} \times 9 \times 6 \times 3$$

$$= \frac{9\times 6\times 3\times 6}{3!}\left(\frac{ig}{3!}\right)^3 \int d^4x\, d^4x'\, d^4x''\, D_{1x}\, D_{2x'}\, D_{3x''}\, D_{xx'}\, D_{x'x''}\, D_{xx''}$$

$$= \frac{\cancel{9}^{\,3}\times\cancel{6}\times\cancel{3}\times\cancel{6}}{\cancel{6}\times\cancel{6}\times\cancel{6}\times{}_2\cancel{6}\ \ 2}(ig)^3 \int ---$$

$$= \frac{3}{4}(ig)^3 \int d^4x\, d^4x'\, d^4x''\, D_{1x}\, D_{2x'}\, D_{3x''}\, D_{xx'}\, D_{x'x''}\, D_{xx''}$$

(d) We want to evaluate $\int d^4x\, d^4x'\, d^4x''\, D_{1x}\, D_{2x'}\, D_{3x''}\, D_{xx'}\, D_{x'x''}\, D_{xx''}$

$$D_{1x} = \int \frac{d^4k}{(2\pi)^4} \frac{i e^{ik_1(x_1 - x)}}{k_1^2 - e\, i\varepsilon}, \quad \text{Let } \int \frac{d^4k}{(2\pi)^4} \text{ be implicit,}$$

we write $D_{1x} = \dfrac{i e^{ik_1(x_1 - x)}}{k_1^2 + i\varepsilon}$ for simplicity,

$D_{1x} D_{2x'} D_{3x''} D_{xx'} D_{x'x''} D_{xx''}$

$$= \frac{i e^{ik_1(x_1-x)}}{k_1^2 + i\varepsilon} \frac{i e^{ik_2(x_2-x')}}{k_2^2 + i\varepsilon} \frac{i e^{ik_3(x_3-x'')}}{k_3^2 + i\varepsilon} \frac{i e^{ik_4(x-x')}}{k_4^2 + i\varepsilon} \frac{i e^{ik_5(x'-x'')}}{k_5^2 + i\varepsilon} \frac{i e^{ik_6(x-x'')}}{k_6^2 + i\varepsilon}$$

$$= \frac{(i)^6\, e^{ix(k_6 + k_4 - k_1)}\, e^{ix'(k_5 - k_4 - k_2)}\, e^{ix''(-k_6 - k_5 - k_3)}\, e^{ik_1x_1}\, e^{ik_2x_2}\, e^{ik_3x_3}}{(\cdots)(\cdots)(\cdots)\cdots(\cdots)}$$

$$= \frac{(i)^6\, \delta(k_6 + k_4 - k_1)\, \delta(k_5 - k_4 - k_2)\, \delta(-k_6 - k_5 - k_3)\, e^{ik_1x_1}\, e^{ik_2x_2}\, e^{ik_3x_3}}{(k_1^2 + i\varepsilon)(k_2^2 + i\varepsilon)(k_3^2 + i\varepsilon)(k_4^2 + i\varepsilon)(k_5^2 + i\varepsilon)(k_6^2 + i\varepsilon)}$$

Integrate k_6, so imposing $k_6 = k_1 - k_4$.

$$\frac{(i)^6\, \delta(k_5 - k_4 - k_2)\, \delta(k_4 - k_1 - k_5 - k_3)\, e^{ik_1x_1} e^{ik_2x_2} e^{ik_3x_3}}{(k_1^2 + i\varepsilon)(k_2^2 + i\varepsilon)(\cdots) \cdots [(k_1 - k_4)^2 + i\varepsilon]}$$

Integrate over k_5, imposing $k_5 = k_4 + k_2$.

$$\frac{(i)^6\, \delta(-k_1 - k_2 - k_3)\, e^{ik_1x_1} e^{ik_2x_2} e^{ik_3x_3}}{(k_1^2 + i\varepsilon)(k_2^2 + i\varepsilon)(k_3^2 + i\varepsilon)(k_4^2 + i\varepsilon)[(k_4 + k_2)^2 + i\varepsilon][(k_1 - k_4)^2 + i\varepsilon]}$$

Applying LSZ, attach $\left[-i\int d^4x_1\, e^{-ip_1x_1}\, p_1^2\right]\left[-i\int d^4x_2\, e^{+ip_2x_2}\, p_2^2\right]\left[-i\int d^4x_3 \cdots\right]$

$$\Rightarrow (i)^6(-i)^3 \int d^4x_1\, d^4x_2\, d^4x_3\, \frac{e^{ix_1(k_1 - p_1)}\, e^{ix_2(k_2 + p_2)}\, e^{ix_3(k_3 + p_3)}\, p_1^2\, p_2^2\, p_3^2}{(k_1^2 + i\varepsilon)(k_2^2 + i\varepsilon)(k_3^2 + i\varepsilon)(k_4^2 + i\varepsilon)[(k_4 + k_2)^2 + i\varepsilon][(k_1 - k_4)^2 + i\varepsilon]} \times \delta(-k_1 - k_2 - k_3)$$

$$= (i)^6(-i)^3 \frac{1}{(k_4^2 + i\varepsilon)[(k_4 - p_2)^2 + i\varepsilon][(p_1 - k_4)^2 + i\varepsilon]} \times \delta(-p_1 + p_2 + p_3)$$

Letting k_q be denoted $k \equiv k_q$, then we have

$$\int \frac{d^4k}{(2\pi)^3} \frac{i}{k^2+i\varepsilon} \frac{i}{(k-p_2)^2+i\varepsilon} \frac{i}{(p_1-k)^2+i\varepsilon} \times \delta(p_2+p_3-p_1)$$

Attaching back $\frac{3}{4}(ig)^3$, we have

$$(-1)\frac{3}{4}g^3 \int \frac{d^4k}{(2\pi)^3} \frac{i}{k^2+i\varepsilon} \frac{i}{(k-p_2)^2+i\varepsilon} \frac{i}{(p_1-k)^2+i\varepsilon} \times \delta(p_2+p_3-p_1)$$

Davidson Chen

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