Schwartz 25.2

(25.20): $\frac{1}{2N}\delta^{ab} + \frac{1}{2} d^{abc} T^c + \frac{1}{2} i f^{abc} T^c$

$$= \frac{1}{2}\left(\frac{1}{N} tr(T^a T^b) + tr(T^a\{T^b, T^c\})T^c + [T^a, T^b]\right) = T^a T^b ?$$

$$T^a T^b = T^a T^c \delta^{cb} = 2 tr(T^b T^c) T^a T^c$$

trace   symmetric traceless   antisymmetric

$$T^a T^b = \frac{1}{2}\{T^a, T^b\} + \frac{1}{2}[T^a, T^b]$$

$$= \frac{1}{2N}\delta^{ab} + \underbrace{\left(\frac{1}{2}\{T^a, T^b\} - \frac{1}{2N}\delta^{ab}\mathbb{1}\right)}_{\text{WTS equals } \frac{1}{2} d^{abc} T^c} + \frac{1}{2} i f^{abc} T^c$$

$\boxed{\text{WTS: } d^{abe} T^e = \{T^a, T^b\} - \frac{1}{N}\delta^{ab}\mathbb{1} \text{ is satisfied by } d^{abc} = 2 tr[T^a\{T^b, T^c\}]}$

multiply each by T^c and take trace:

$$\frac{d^{abc}}{2} = tr[\{T^a, T^b\} T^c]$$

$$\boxed{d^{abc} = 2 tr[T^c\{T^a, T^b\}]}$$

(25.21): $\mathrm{tr}(T^aT^bT^c) = \frac{1}{2}\mathrm{tr}(T^aT^bT^c + T^cT^aT^b)$

$$= \frac{1}{2}\mathrm{tr}(T^aT^bT^c + T^c([T^a,T^b] + T^bT^a))$$

$$= \frac{1}{2}\mathrm{tr}(T^aT^bT^c + T^aT^cT^b + if^{abd}T^cT^d)$$

$$= \frac{1}{2}\mathrm{tr}(T^a\{T^b,T^c\}) + if^{abd}\frac{1}{2}\frac{1}{2}\delta^{cd})$$

$$= \boxed{\frac{1}{4}(d^{abc} + if^{abc})}$$

(25.22): use $T^aT^b = \frac{1}{2N}\delta^{ab} + \frac{1}{2}d^{abc}T^c + \frac{1}{2}if^{abc}T^c$

$$= \frac{1}{2}\left(\frac{\delta^{ab}}{N} + (d^{abc} + if^{abc})T^c\right)$$

$$\mathrm{tr}(T^aT^bT^cT^d) = \frac{1}{4}\mathrm{tr}\Big(\frac{1}{N^2}\delta^{ab}\delta^{cd} + \frac{1}{N}\delta^{ab}(d^{cde} + if^{cde})T^e$$
$$+ \frac{1}{N}\delta^{cd}(d^{abg} + if^{abg})T^g$$
$$+ (d^{cde} + if^{cde})(d^{abg} + if^{abg})T^eT^g\Big)$$

$$= \frac{1}{4}\frac{1}{N^2}\mathrm{tr}(\delta^{ab}\delta^{cd}) + \frac{1}{4}(d^{cde} + if^{cde})(d^{abg} + if^{abg})\mathrm{tr}(T^eT^g)$$

$$= \boxed{\frac{1}{4N}\delta^{ab}\delta^{cd} + \frac{1}{8}(d^{cde} + if^{cde})(d^{abe} + if^{abe})}$$

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