

Schritt 25.1 (23.67): $\delta A_\mu^a = \frac{1}{g} \partial_\mu \alpha^a - f^{abc} \alpha^b A_\mu^c$

$$-4 \mathcal{L}_m = (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c)^2 = (F_{\mu\nu}^a)^2$$

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$$

$$\delta F_{\mu\nu}^a = \partial_\mu (\delta A_\nu^a) - \partial_\nu (\delta A_\mu^a) + g f^{abc} [\delta A_\mu^b A_\nu^c + A_\mu^b (\delta A_\nu^c)]$$

$$= \partial_\mu (-f^{abc} \alpha^b A_\nu^c) - \partial_\nu (-f^{abc} \alpha^b A_\mu^c) + g f^{abc} \left[\left(\frac{1}{g} \partial_\mu \alpha^b - f^{bde} \alpha^d A_\mu^e \right) A_\nu^c + A_\mu^b \left(\frac{1}{g} \partial_\nu \alpha^c - f^{cde} \alpha^d A_\nu^e \right) \right]$$

$$= f^{abc} [\partial_\mu \alpha^b A_\nu^c - \partial_\nu \alpha^b A_\mu^c + \alpha^b \partial_\mu A_\nu^c - \alpha^b \partial_\nu A_\mu^c] + g f^{abc} \left[\frac{1}{g} (\partial_\mu \alpha^b) A_\nu^c - f^{bde} \alpha^d A_\mu^e A_\nu^c + \frac{1}{g} (\partial_\nu \alpha^c) A_\mu^b - f^{cde} \alpha^d A_\nu^e A_\mu^b \right]$$

$$= f^{abc} \alpha^b (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) + f^{abc} [\cancel{\alpha^b \partial_\mu A_\nu^c} - \cancel{\alpha^b \partial_\nu A_\mu^c}] + f^{abc} (\cancel{\alpha^b \partial_\mu A_\nu^c} + \cancel{\alpha^b \partial_\nu A_\mu^c}) - \alpha^d g f^{abc} (f^{bde} A_\mu^e A_\nu^c + f^{cde} A_\nu^e A_\mu^b)$$

$$= \alpha^b f^{abc} (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) - \alpha^d g [f^{ace} f^{edb} A_\mu^b A_\nu^c + f^{abe} f^{edc} A_\mu^b A_\nu^c]$$

$$= -\alpha^b f^{abc} (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) - g \alpha^d [f^{ace} f^{edb} + f^{abe} f^{edc}] A_\mu^b A_\nu^c$$

$$= -\alpha^b f^{abc} (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) + g \alpha^d [f^{ade} f^{ecb}] A_\mu^b A_\nu^c$$

$$= \alpha^b [-f^{abc} (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) + g f^{abc} f^{ced} A_\mu^d A_\nu^e]$$

$$= -\alpha^b f^{abc} [\partial_\mu A_\nu^c - \partial_\nu A_\mu^c + g f^{cde} A_\mu^d A_\nu^e]$$

$$= i \alpha^b (i f^{abc}) (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c + g f^{cde} A_\mu^d A_\nu^e)$$

$$\Rightarrow \boxed{(i \alpha^b) (i f^{abc}) F_{\mu\nu}^c = \delta F_{\mu\nu}^a}$$

$$\begin{aligned} f(F_{\mu\nu}^a F^{\mu\nu a}) &= 2 (\delta F_{\mu\nu}^a) F^{\mu\nu a} \\ &= -2 \alpha^b f^{abc} F_{\mu\nu}^c F^{\mu\nu a} \\ &= -\alpha^b f^{abc} (F_{\mu\nu}^c F^{\mu\nu a} - F_{\mu\nu}^a F^{\mu\nu c}) \end{aligned}$$

$$= \boxed{0}$$

Chang Chen

Jan. 7, 2026