Schwartz 6.2 progress

$$\tilde V(p)=\frac{e_R^2}{p^2}\left\{1+\frac{e_R^2}{2\pi^2}\int_0^1 dx\,x(1-x)\ln\left[1-\frac{p^2}{m^2}x(1-x)\right]\right\}.$$

$$\tilde V(\vec y)=\int\frac{d^3\vec p}{(2\pi)^3}e^{i\vec y\vec p}\frac{e_R^2}{|\vec p|^2}\left\{1+\frac{e_R^2}{2\pi^2}\int_0^1 dx\,x(1-x)\ln\left[1-\frac{p^2}{m^2}x(1-x)\right]\right\}.$$

$$=\frac{e_R^2}{(2\pi)^3}\int dp\,d\cos\theta\,d\phi\,e^{irp\cos\theta}\left\{\cancel{1+\frac{e_R^2}{2\pi^2}\int_0^1}dx\ \cdots\right\}.$$

$$=\frac{e_R^2}{(2\pi)^2}\int dp\,\frac{1}{irp}\left(e^{irp}-e^{-irp}\right)\left\{\cancel{1+e_R^2}\ 1+\cdots\right\}.$$

$$\int\frac{dp}{irp}\left(e^{irp}-e^{-irp}\right)=\frac{1}{2ir}\int_{-\infty}^{\infty}dp\,\frac{1}{p+i\varepsilon}\left(e^{irp}-e^{-irp}\right)=\frac{1}{\cancel{2}ir}2\pi i(-1)$$

$$=-\frac{\pi}{r}.$$

$$\Rightarrow\ =-\frac{e_R^2}{(2\pi)^2}\frac{\pi}{r}+\frac{e_R^2}{(2\pi)^2}\int dp\,\frac{1}{irp}\left(e^{irp}-e^{-irp}\right)\frac{e_R^2}{2\pi^2}\int_0^1 dx\,x(1-x)\ln\left[1-\frac{p^2}{m^2}x(1-x)\right]$$

$$=-\frac{e_R^2}{4\pi r}+\frac{e_R^4}{(2\pi)^2 r}\int\frac{dp}{p}\sin(rp)\frac{1}{\pi^2}\int_0^1 dx\,x(1-x)\ln\left[1-\frac{p^2}{m^2}x(1-x)\right]$$

$$=\boxed{-\frac{e_R^2}{4\pi r}+\frac{e_R^4}{4\pi^4 r}\int_0^\infty\frac{dp}{p}\sin(rp)\underbrace{\int_0^1 dx\,x(1-x)\ln\left[1-\frac{p^2}{m^2}x(1-x)\right]}}$$

$$\int_0^1 dx\,x(1-x)\int_0^\infty\frac{dp}{p}\sin(rp)\ln\left[1-\frac{p^2}{m^2}x(1-x)\right].$$

$$=\int_0^1 dx\,x(1-x)\underbrace{\int_0^\infty\frac{dp}{p}\sin(rp)\ln\left[1-ap^2\right]}_{\Downarrow}\qquad a\equiv\frac{x(1-x)}{m^2}.$$

$$\frac12\int_{-\infty}^{\infty}\frac{dp}{p}\sin(rp)\ln|1-ap^2|\qquad a\geq 0.$$

Consider expansion of ln: $\quad I(a,p)\equiv\int_0^\infty\frac{dp}{p}\sin(rp)\ln|1-ap^2|$

$\ln(1-x)=\sum$

$$\frac{\partial I}{\partial a}=\int_0^\infty\frac{dp}{p}\frac{\sin(rp)}{(1-ap^2)}(-p^2)=-\frac12\int_{-\infty}^{\infty}dp\,\frac{p\sin(rp)}{(1-ap^2)}$$