Schwartz
10.2

(a) Consider $\gamma_3 \gamma^{\pm} V$

$$= \gamma_3 [\gamma_1 \pm i\gamma_2] V$$

$$\gamma_3\gamma_1 - \gamma_1\gamma_3 = i\gamma_2, \qquad \gamma_3\gamma_2 - \gamma_2\gamma_3 = -i\gamma_1$$

$$\Rightarrow \gamma_3\gamma_1 = i\gamma_2 + \gamma_1\gamma_3, \qquad \pm i\gamma_3\gamma_2 = \pm i[-i\gamma_1 + \gamma_2\gamma_3]$$

$$\gamma_3\gamma^{\pm} V = \gamma_3[\gamma_1 \pm i\gamma_2] V$$

$$= [\gamma_3\gamma_1 \pm i\gamma_3\gamma_2] V$$

$$= [i\gamma_2 \pm \gamma_1 \pm i\gamma_2\gamma_3 + \gamma_1\gamma_3] V$$

$$= \{\pm\gamma_1 + i\gamma_2 + [\gamma_1 \pm i\gamma_2]\gamma_3\} V$$

$$= \cancel{\pm\gamma^{\pm}} \{\pm\gamma^{\pm} + \gamma^{\pm}\lambda\} V$$

$$= \boxed{[\lambda \pm 1]\gamma^{\pm} V}$$

(b) $\gamma_3 \gamma^{\pm} V = (\lambda \pm 1) \gamma^{\pm} V$ implies $\gamma^{\pm} V$ is an eigenvector of γ_3, or zero.

Since γ are finite (n) dimensional, there are n eigenvectors, suppose $\gamma_3 V_{max} = \lambda_{max} V_{max}$ is the one with maximum eigenvalue. Then by contradiction $\gamma^+ V_{max} = 0$, because otherwise $\gamma_3 \gamma^+ V_{max} = (\lambda_{max} + 1) \gamma^+ V_{max}$ and $\gamma^+ V_{max}$ has eigenvalue $\lambda_{max} + 1$.

The same argument goes for V_{min}.

(d) 5-dimensional, $5-1=4$, $4/2=2$. $J=2$.

$$\Rightarrow J_3 = \begin{bmatrix} 2 & & & & \\ & 1 & & & \\ & & 0 & & \\ & & & -1 & \\ & & & & -2 \end{bmatrix}$$

$$J^+ = \begin{bmatrix} 0 & 1 & & & \\ & 0 & 1 & & \\ & & 0 & 1 & \\ & & & 0 & 1 \\ & & & & 0 \end{bmatrix} \quad J^- = \begin{bmatrix} 0 & & & & \\ 1 & 0 & & & \\ & 1 & 0 & & \\ & & 1 & 0 & \\ & & & 1 & 0 \end{bmatrix}$$

$J^+ + J^- = 2J_1 \Rightarrow$

$$J_1 = \frac{1}{2}\begin{bmatrix} 0 & 1 & & & \\ 1 & 0 & 1 & & \\ & 1 & 0 & 1 & \\ & & 1 & 0 & 1 \\ & & & 1 & 0 \end{bmatrix}$$

$J^+ - J^- = 2iJ_2 \Rightarrow$

$$J_2 = \frac{1}{2i}\begin{bmatrix} 0 & 1 & & & \\ -1 & 0 & 1 & & \\ & -1 & 0 & 1 & \\ & & -1 & 0 & 1 \\ & & & -1 & 0 \end{bmatrix}$$

$$\Rightarrow J_3 = \begin{bmatrix} 2 & & & & \\ & 1 & & & \\ & & 0 & & \\ & & & -1 & \\ & & & & -2 \end{bmatrix}, \quad J_1 = \frac{1}{2}\begin{bmatrix} 0 & 1 & & & \\ 1 & 0 & 1 & & \\ & 1 & 0 & 1 & \\ & & 1 & 0 & 1 \\ & & & 1 & 0 \end{bmatrix}, \quad J_2 = \frac{1}{2i}\begin{bmatrix} 0 & 1 & & & \\ -1 & 0 & 1 & & \\ & -1 & 0 & 1 & \\ & & -1 & 0 & 1 \\ & & & -1 & 0 \end{bmatrix}$$