

Griffiths, 8.2

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{5|M|^2}{4E^2} \frac{|\vec{P}_3|}{|\vec{P}_1|}$$

$$|\vec{P}_3|^2 = \frac{E^2}{c^2} - M^2 c^2, \quad |\vec{P}_1|^2 = \frac{E^2}{c^2} - m^2 c^2$$

$$\frac{|\vec{P}_3|}{|\vec{P}_1|} = \sqrt{\frac{E^2/c^2 - M^2 c^2}{E^2/c^2 - m^2 c^2}}$$

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{5|M|^2}{4E^2} \frac{\sqrt{E^2/c^2 - M^2 c^2} \sqrt{1 - m^2 c^4/E^2}}{\sqrt{E^2/c^2 - m^2 c^4/E^2}}$$

this formula is θ -independent except $|M|^2$, so we can independently integrate $|M|^2$ over $d\Omega = \sin\theta d\theta d\phi$.

$$\langle |M|^2 \rangle = Q_e^2 g_e^4 \left\{ 1 + \left(\frac{mc^2}{E}\right)^2 + \left(\frac{Mc^2}{E}\right)^2 + \left[1 - \left(\frac{mc^2}{E}\right)^2\right] \left[1 - \left(\frac{Mc^2}{E}\right)^2\right] \cos^2\theta \right\}$$

$$\text{letting } A = \frac{mc^2}{E}, \quad B = \frac{Mc^2}{E}$$

$$\langle |M|^2 \rangle = Q_e^2 g_e^4 \left\{ 1 + A^2 + B^2 + (1 - A^2)(1 - B^2) \cos^2\theta \right\}$$

$$\int \langle |M|^2 \rangle d\theta \sin\theta d\phi = Q_e^2 g_e^4 \int [1 + A^2 + B^2 + (1 - A^2)(1 - B^2) \cos^2\theta] d\Omega$$

$$= Q_e^2 g_e^4 4\pi \left[\int (1 + A^2 + B^2) \sin\theta d\theta + \int (1 - A^2)(1 - B^2) \cos^2\theta \sin\theta d\theta \right]$$

$$\int_0^\pi (1+A^2+B^2) \sin\theta d\theta = 1+A^2+B^2,$$

$$\int_0^\pi (1-A^2)(1-B^2) \cos^2\theta \sin\theta d\theta = \frac{1}{3}(1-A^2)(1-B^2).$$

$$1+A^2+B^2 + \frac{1}{3}(1-A^2)(1-B^2) = \frac{4}{3} + \frac{2}{3}A^2 + \frac{2}{3}B^2 + \frac{1}{3}A^2B^2$$

$$= \frac{4}{3} \left[1 + \frac{1}{2}A^2 + \frac{1}{2}B^2 + \frac{1}{4}A^2B^2 \right]$$

$$= \frac{4}{3} \left[1 + \frac{1}{2}A^2 \right] \left[1 + \frac{1}{2}B^2 \right]$$

$$\Rightarrow \int \langle M^2 \rangle d\Omega = 4\pi Q^2 g_e^4 \frac{4}{3} \left[1 + \frac{1}{2}A^2 \right] \left[1 + \frac{1}{2}B^2 \right],$$

$$\sigma = \left(\frac{\hbar c}{8\pi} \right)^2 \frac{5}{E^2} \frac{4\pi Q^2 g_e^4}{3} \frac{\sqrt{1-B^2}}{\sqrt{1-A^2}} \left[1 + \frac{1}{2}A^2 \right] \left[1 + \frac{1}{2}B^2 \right].$$

$$\left[= \left(\frac{\hbar c}{8\pi E} \right)^2 \frac{5}{3} \frac{4\pi Q^2 g_e^4}{\sqrt{1-(m_e^2/E)^2}} \left[1 + \frac{1}{2} \left(\frac{m_e^2}{E} \right)^2 \right] \left[1 + \frac{1}{2} \left(\frac{m_e^2}{E} \right)^2 \right] \right]$$

Putting proportionality factors into d shall give 8.5