

Griffiths

9.5. (a) $f^{\alpha\beta\gamma}$ antisymmetric means for it to be non zero we need $\alpha \neq \beta \neq \gamma$. This gives $8 \times 7 \times 6$ values.

Now ~~any permutation~~

permuting $f^{\alpha\beta\gamma}$ introduces a $+1$ or -1 factor, so we need to divide by # of permutations

$$\frac{8 \times 7 \times 6}{1 \times 2 \times 3} = 56.$$

$$\begin{aligned} (b) \quad [\lambda^1, \lambda^2] &= \begin{bmatrix} [e_1, e_2] & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 2i e_3 & 0 \\ 0 & 0 \end{bmatrix} \\ &= \begin{bmatrix} 2i & 0 \\ 0 & -2i \\ 0 & 0 \end{bmatrix} \end{aligned}$$

The form above indicates that $[\lambda^1, \lambda^2]$ is only linearly combination of λ^3 , which makes it obvious that $f^{12\gamma} = 0$ for $\gamma \neq 3$.

$$\begin{aligned} (c) \quad [\lambda^1, \lambda^3] &= \begin{bmatrix} [e_1, e_3] & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -2i e_2 & 0 \\ 0 & 0 \end{bmatrix} = -2i \lambda^2 \\ &\Rightarrow f^{13\gamma} = \begin{cases} 0 & \text{if } \gamma \neq 2 \\ 1 & \text{if } \gamma = 2 \end{cases} \end{aligned}$$

$$[\lambda^4, \lambda^5] = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix} = 2i \left[\frac{\lambda^3}{2} + \frac{\sqrt{3}}{2} \lambda^8 \right]$$

$$\Rightarrow f^{453} = \frac{1}{2}, \quad f^{458} = \frac{\sqrt{3}}{2}.$$