

Griffiths.

8.1 (a), (b), (c) are easily seen, we do (d) below.

$$\langle M^2 \rangle = \frac{Q^2 g_e^4}{(P_1 + P_2)^4} \left[(P_1 \cdot P_3)(P_2 \cdot P_4) + (P_1 \cdot P_4)(P_2 \cdot P_3) + (mc)^2 (P_3 \cdot P_4) + (Mc)^2 (P_3 \cdot P_2) + 2(mc)^2 (Mc)^2 \right]$$

$$(P_1 + P_2)^4 = \left(\frac{E}{c} \right)^4$$

$$(P_1 \cdot P_3) = \frac{E^2}{4c^2} - \vec{P}_1 \cdot \vec{P}_3, \quad (P_1 \cdot P_4) = \frac{E^2}{4c^2} + \vec{P}_1 \cdot \vec{P}_3$$

$$(P_2 \cdot P_4) = \frac{E^2}{4c^2} - \vec{P}_1 \cdot \vec{P}_3, \quad (P_2 \cdot P_3) = \frac{E^2}{4c^2} + \vec{P}_1 \cdot \vec{P}_3$$

$$(P_3 \cdot P_4) = \frac{E^2}{4c^2} + |\vec{P}_3|^2, \quad (P_1 \cdot P_2) = \frac{E^2}{4c^2} + |\vec{P}_1|^2$$

$$\langle M^2 \rangle = \left(Q^2 g_e^4 \right) \left(\frac{8c^4}{E^4} \right) \left[\left[\frac{E^2}{4c^2} \right]^2 + \left[\vec{P}_1 \cdot \vec{P}_3 \right]^2 + \left[\frac{E^2}{4c^2} \right]^2 + \left[\vec{P}_1 \cdot \vec{P}_3 \right]^2 + (mc)^2 \left[\frac{E^2}{4c^2} + |\vec{P}_3|^2 \right] + (Mc)^2 \left[\frac{E^2}{4c^2} + |\vec{P}_1|^2 \right] + 2(mc)^2 (Mc)^2 \right]$$

$$= (Q^2 g_e^4) \left[\frac{8c^4}{E^4} \right] \left\{ \frac{E^4}{8c^4} + 2(\vec{P}_1 \cdot \vec{P}_3)^2 + \frac{E^2 m^2}{4c^4} + \frac{m^2 c^2 |\vec{P}_3|^2}{4c^4} + \frac{E^2 M^2}{4c^4} + \frac{M^2 c^2 |\vec{P}_1|^2}{4c^4} + 2m^2 M^2 c^4 \right\}$$

$$= Q^2 g_e^4 \left[1 + \frac{16c^4 (\vec{P}_1 \cdot \vec{P}_3)^2}{E^4} + \frac{2m^2 c^4}{E^2} + \frac{8m^2 c^6 |\vec{P}_3|^2}{E^4} + \frac{2M^2 c^4}{E^2} + \frac{8M^2 c^6 |\vec{P}_1|^2}{E^4} + \frac{16m^2 M^2 c^8}{E^4} \right]$$

$$= Q^2 g_e^4 \left[1 + \frac{2m^2 c^4}{E^2} + \frac{2M^2 c^4}{E^2} + \frac{1}{E^4} [---] \right]$$

$$4(\vec{p}_1 \cdot \vec{p}_3)^2 = \cos^2 \theta |\vec{p}_1|^2 |\vec{p}_3|^2 c^4, \quad |\vec{p}_1|^2 c^2 = \frac{E^2}{4} - m^2 c^4, \quad |\vec{p}_3|^2 c^2 = \frac{E^2}{4} - M^2 c^4.$$

$$[---] = [16 c^4 (\vec{p}_1 \cdot \vec{p}_3)^2 + 8 m^2 c^4 |\vec{p}_3|^2 + 8 M^2 c^4 |\vec{p}_1|^2 + 16 m^2 M^2 c^8]$$

$$= \left[16 \cos^2 \theta \left[\frac{E^2}{4} - m^2 c^4 \right] \left[\frac{E^2}{4} - M^2 c^4 \right] + 8 m^2 c^4 \left[\frac{E^2}{4} - M^2 c^4 \right] \right. \\ \left. + 8 M^2 c^4 \left[\frac{E^2}{4} - m^2 c^4 \right] + 16 m^2 M^2 c^8 \right]$$

$$= \left[\cos^2 \theta \left[E^2 - \frac{m^2 c^4}{4} \right] \left[E^2 - \frac{M^2 c^4}{4} \right] + 2 m^2 c^4 E^2 + 2 M^2 c^4 E^2 \right]$$

$$\frac{1}{E^4} [---] = \cos^2 \theta \left[1 - \frac{m^2 c^4}{4 E^2} \right] \left[1 - \frac{M^2 c^4}{4 E^2} \right] + \frac{2 m^2 c^4}{E^2} + \frac{2 M^2 c^4}{E^2}$$

$$\Rightarrow \langle M^2 \rangle = Q^2 g_e^4 \left\{ 1 + \frac{4 m^2 c^4}{E^2} + \frac{4 M^2 c^4}{E^2} + \cos^2 \theta \left[1 - \frac{m^2 c^4}{4 E^2} \right] \left[1 - \frac{M^2 c^4}{4 E^2} \right] \right\}$$

$$= Q^4 g_e^4 \left[1 + \frac{2 m^2 c^4}{E^2} + \left(\frac{2 m c^2}{E} \right)^2 + \left(\frac{2 M c^2}{E} \right)^2 + \cos^2 \theta \left[1 - \left(\frac{m c^2}{2 E} \right)^2 \right] \left[1 - \left(\frac{M c^2}{2 E} \right)^2 \right] \right]$$

This is equivalent to (8.4) upto a redefinition of $E \rightarrow 2E$.