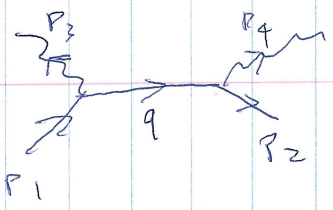
Griffiths.

7.25. Determine the total amplitude for pair annihilation $e^+ + e^- \to \gamma + \gamma$.

There are two diagrams.



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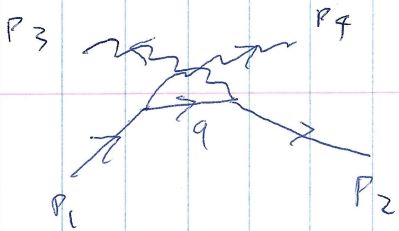

$$\int \frac{d^4q}{(2\pi)^4} \left[\varepsilon_3^{\mu *} i g_e \gamma_\mu u_1\right] \frac{i(\not{q} + mc)}{q^2 - m^2c^2} \left[\varepsilon_4^{\nu *} i g_e \gamma_\nu u_2\right]$$

$$(2\pi)^4 \delta(p_1 - p_3 - q)\,(2\pi)^4 \delta(q - p_2 - p_4)$$

$$= \int -i g_e^2 \left[\varepsilon_3^{\mu *} \gamma_\mu u_1\right] \frac{(\not{q} + mc)}{q^2 - m^2c^2} \left[\varepsilon_4^{\nu *} \gamma_\nu u_2\right] d^4q$$

$$(2\pi)^4 \delta(p_1 - p_3 - q)\,\delta(q - p_2 - p_4)$$

$$= -i g_e^2 \left[\varepsilon_3^{\mu *} \gamma_\mu u_1\right] \frac{(\not{p}_1 - \not{p}_3 + mc)}{(p_1 - p_3)^2 - m^2c^2} \left[\varepsilon_4^{\nu *} \gamma_\nu u_2\right] (2\pi)^4 \delta(p_1 - p_3 - p_2 - p_4)$$

$$= -i g_e^2 \left(\not{\varepsilon}_3^{*} u_1\right) \frac{(\not{p}_1 - \not{p}_3 + mc)}{(p_1 - p_3)^2 - m^2c^2} \left(\not{\varepsilon}_4^{*} u_2\right) (2\pi)^4 \delta(p_1 - p_3 - p_2 - p_4).$$

$$\Rightarrow \mathcal{M} = g_e^2 \left[(\varepsilon_3^* u_1)\right]\left[\frac{(\not{p}_1 - \not{p}_3) + mc}{(p_1 - p_3)^2 - m^2c^2}\right]\left[\not{\varepsilon}_4^* u_2\right]$$

$$+ g_e^2 \left[\varepsilon_4^* u_1\right]\left[\frac{(\not{p}_1 - \not{p}_4) + mc}{(p_1 - p_4)^2 - m^2c^2}\right]\left[\varepsilon_3^* u_2\right].$$

~~the~~ addition because they only differ by exchange of 2 photons.