7.10: (Griffith)

Suppose we pick $i\hbar\gamma^\mu\partial_\mu\psi - mc\psi$ as Dirac's Equation, then setting $\vec{p}=0$, we find (in rest frame),

$$\psi = \begin{pmatrix}\psi_A \\ \psi_B\end{pmatrix} = \begin{pmatrix} e^{\frac{imc^2}{\hbar}t}\,\psi_A(0) \\ e^{-\frac{imc^2}{\hbar}t}\,\psi_B(0)\end{pmatrix}$$

This is equivalent to that found in section 7.2 with ψ_A, ψ_B swapped.

To verify this, apply basis and 1-form transformation T to the Dirac equation:

$$T^{-1}(i\hbar\gamma^\mu\partial_\mu - mc)T\psi = 0$$

with $T = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$ doing basis swapping, we find

$$T^{-1}\begin{pmatrix}1 & \\ & -1\end{pmatrix}T = \begin{pmatrix}-1 & \\ & 1\end{pmatrix}$$

$$T^{-1}\begin{pmatrix} & \vec{\sigma} \\ -\vec{\sigma} & \end{pmatrix}T = \begin{pmatrix} & -\vec{\sigma} \\ \vec{\sigma} & \end{pmatrix}$$

That is,

$$T^{-1}\gamma^\mu T = -\gamma^\mu,$$

giving us

$$T^{-1}(i\hbar\gamma^\mu\partial_\mu - mc)T\psi$$

$$= (-i\hbar\gamma^\mu\partial_\mu - mc)\psi.$$

That is, the two forms of Dirac Equation are related by a basis swap.