

Qn 7-10

$$4.28. (a) \mu_a = \langle \pi^+ p | \pi^+ p \rangle$$

$$= \left(\langle 11 | K_{\frac{1}{2} \frac{1}{2}} | \right) \left(| 11 \rangle | \frac{1}{2} \frac{1}{2} \rangle \right)$$

$$= \langle \frac{3}{2} \frac{3}{2} | \frac{3}{2} \frac{3}{2} \rangle = \mu_3.$$

$$\mu_b = \langle \pi^0 p \rightarrow \pi^0 p \rangle.$$

$$= \frac{2}{3} \langle \frac{3}{2} \frac{1}{2} | \frac{3}{2} \frac{1}{2} \rangle + \frac{1}{3} \langle \frac{1}{2} \frac{1}{2} | \frac{1}{2} \frac{1}{2} \rangle$$

$$= \frac{2}{3} \mu_3 + \frac{1}{3} \mu_1.$$

$$\mu_c \Leftrightarrow \pi^- p \rightarrow \pi^- p$$

$$= \frac{1}{3} \langle \frac{3}{2} -\frac{1}{2} | \frac{3}{2} -\frac{1}{2} \rangle + \frac{2}{3} \langle \frac{1}{2} -\frac{1}{2} | \frac{1}{2} -\frac{1}{2} \rangle$$

$$= \frac{1}{3} \mu_3 + \frac{2}{3} \mu_1.$$

$$\mu_d \Leftrightarrow \pi^+ n \rightarrow \pi^+ n$$

$$= \frac{1}{3} \langle \frac{3}{2} \frac{1}{2} | \frac{3}{2} \frac{1}{2} \rangle + \frac{2}{3} \langle \frac{1}{2} \frac{1}{2} | \frac{1}{2} \frac{1}{2} \rangle$$

$$= \frac{1}{3} \mu_3 + \frac{2}{3} \mu_1.$$

$$\mu_e = \frac{2}{3} \langle \frac{3}{2} -\frac{1}{2} | \frac{3}{2} -\frac{1}{2} \rangle + \frac{1}{3} \langle \frac{1}{2} -\frac{1}{2} | \frac{1}{2} -\frac{1}{2} \rangle$$

$$= \frac{2}{3} \mu_3 + \frac{1}{3} \mu_1.$$

$$\mu_f = \mu_3.$$

$$\mu_g = \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} \mu_3 - \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} \mu_1 = \frac{\sqrt{2}}{3} [\mu_3 - \mu_1]$$

$$\begin{aligned} \mu_h &= \sqrt{\frac{2}{3}} \sqrt{\frac{1}{3}} \left\langle \frac{3}{2} \frac{1}{2} \middle| \frac{3}{2} \frac{1}{2} \right\rangle - \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} \left\langle \frac{1}{2} \frac{1}{2} \middle| \frac{1}{2} \frac{1}{2} \right\rangle \\ &= \frac{\sqrt{2}}{3} [\mu_3 - \mu_1] \end{aligned}$$

$$\begin{aligned} \mu_i &= \frac{1}{\sqrt{3}} \sqrt{\frac{2}{3}} \left\langle \frac{3}{2} - \frac{1}{2} \middle| \frac{3}{2} - \frac{1}{2} \right\rangle - \sqrt{\frac{2}{3}} \frac{1}{\sqrt{3}} \left\langle \frac{1}{2} - \frac{1}{2} \middle| \frac{1}{2} - \frac{1}{2} \right\rangle \\ &= \frac{\sqrt{2}}{3} [\mu_3 - \mu_1] \end{aligned}$$

$$\mu_j = \frac{\sqrt{2}}{3} [\mu_3 - \mu_1]$$

(b). $\sigma_g \neq \sigma_h \neq \sigma_i \neq \sigma_j$

$$\begin{aligned} \sigma_a : \sigma_b : \sigma_c : \sigma_d : \sigma_e : \sigma_f &= 9|\mu_3|^2 : |2\mu_3 + \mu_1|^2 : |\mu_3 + 2\mu_1|^2 \\ & : | \mu_3 + 2\mu_1 |^2 : | \mu_3 + \mu_1 |^2 : 9|\mu_3|^2 \end{aligned}$$

or more succinctly,

$$\sigma_b = \sigma_e, \quad \sigma_a = \sigma_f, \quad \sigma_c = \sigma_d, \quad \sigma_g = \sigma_h = \sigma_i = \sigma_j$$

$$\boxed{\sigma_b : \sigma_a : \sigma_c : \sigma_g = \frac{9|\mu_3|^2}{|2\mu_3 + \mu_1|^2} : 9|\mu_3|^2 : |\mu_3 + 2\mu_1|^2 : |\mu_3 - \mu_1|^2}$$

(d) In $\mu_3 \gg \mu_1$ approx we have

$$\boxed{\sigma_b : \sigma_a : \sigma_c : \sigma_g \approx 4 : 9 : 1 : 2}$$