Townsend (2.41)

$$\dot{R}^2 = A R^{-1} - B, \qquad A = (1-\varepsilon^2) R_{max}, \quad B = (1-\varepsilon^2)$$

$$\Rightarrow 2\cancel{\dot{R}}\ddot{R} = -A R^{-2} \cancel{\dot{R}}$$

$$\boxed{\ddot{R} = -\frac{A}{2} R^{-2}}$$

Solving via ansatz $R = \alpha \tau^{\lambda}$

$$\Rightarrow \alpha \lambda (\lambda - 1) \tau^{\lambda - 2} = -\frac{A}{2} \alpha^{-2} \tau^{-2\lambda}$$

$$\lambda - 2 = -2\lambda \Rightarrow \lambda = 2/3$$

$$\Rightarrow \alpha \left(\frac{2}{3}\right)\left(-\frac{1}{3}\right) = -\frac{A}{2} \alpha^{-2}$$

$$\alpha = \left(\frac{9}{4} A\right)^{1/3} = \left[\frac{9}{4}(1-\varepsilon^2) R_{max}\right]^{1/3}$$

Solving $R(\tau^*) = R_{max}$ for τ^*, while $R(0) = 0$

$$R_{max} = \left[\frac{9}{4}(1-\varepsilon^2) R_{max}\right]^{1/3} \tau^{*\,2/3}$$

$$R_{max}^{\cancel{3}\,2} = \frac{9}{4}(1-\varepsilon^2) \cancel{R_{max}}\, \tau^{*\,2}$$

$$\tau^* = \frac{R_{max}}{(1-\varepsilon^2)^{1/2}} \frac{2}{3}$$

Recall $R_{max} = \dfrac{2M}{1-\varepsilon^2} \Rightarrow$ $\boxed{\tau^* = \dfrac{4M}{3} \dfrac{1}{(1-\varepsilon^2)^{3/2}}}$

⇑ A factor within (2.41)

Davidson Cheng

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