Townsend bolded exercise after (2.24).

$$\frac{dQ}{d\lambda} = \dot{Q} = (k^{\mu} p_{\mu})^{\dot{}} = \dot{k}^{\mu} p_{\mu} + k^{\mu} \dot{p}_{\mu}$$

The Lagrangian doesn't contain $x^{\mu}$ terms, so

$$0 = \frac{\partial L}{\partial x^{\mu}} = \frac{d}{d\lambda} \frac{\partial L}{\partial \dot{x}^{\mu}} \propto \frac{d}{d\lambda} p^{\mu} = \dot{p}^{\mu}$$

$$\Rightarrow \frac{dQ}{d\lambda} = \dot{k}^{\mu} p_{\mu}$$

But $\dot{k}^{\mu} = \dot{x}^{\lambda} (D_{\lambda} k^{\mu})$

By defn, $(\mathcal{L}_k g)_{\mu\nu} = 2 D_{\mu} k_{\nu} = 0$ (because $k$ is killing)

$\Rightarrow \dot{x}^{\lambda} (D_{\lambda} k^{\mu}) = 0 \Rightarrow \dot{k}^{\mu} p_{\mu} = 0$ as well.

$\Rightarrow \boxed{\dot{Q} = 0}$

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