Townsend (2.118)

(i) We know $S(r) = r - 2M$ is a null hypersurface in the Schwarzschild metric, in (x, t, θ, ϕ) coords, with $\frac{x^2}{8M} = ~~r - 2M,~~$ ~~this is~~

$S(x) = x^2 \Rightarrow x = 0$ defines a null hypersurface in (x, t, θ, ϕ) coordinates.

Now consider an element l of the null hypersurface $x^2 = 0$: $l \in \{x = 0\}$, it satisfies

$l \cdot l = 0 \Rightarrow l$ orthogonal to l,
$\Rightarrow l$ null by definition of null hypersurface

$U' = -xe^{-kt} = 0$, $V' = xe^{kt} = 0$ are merely elements of $\{x = 0\} \Rightarrow U', V'$ are null, so they are tangent vectors to some null curve $\Rightarrow$ null curve. Specifically, since ~~Rindler x~~ $x = 0$ of Rindler is a killing horizon of $k = \frac{\partial}{\partial t}$, we can identy the tangent.

$$\boxed{l = \frac{\partial}{\partial t} x \text{ is null}}$$