Schutz
7.4

$$\frac{d}{d\tau}p^0 + \Gamma^0_{\alpha\beta}p^\alpha p^\beta = 0.$$

$$\Gamma^0_{\alpha\beta} = \frac{1}{2}g^{0\sigma}\left[g_{\sigma\alpha,\beta} + g_{\sigma\beta,\alpha} - g_{\alpha\beta,\alpha}\right]$$

Demands $\sigma = 0$.

$$g_{\alpha\beta} = \begin{pmatrix} -(1+2\phi) & & & \\ & (1-2\phi) & & \\ & & (1-2\phi) & \\ & & & (1-2\phi) \end{pmatrix}$$

$$\Rightarrow g^{00} = \frac{-1}{1+2\phi}.$$

$$\Gamma^0_{\alpha\beta} = \frac{1}{2}\left[\frac{-1}{1+2\phi}\right]\left[g_{0\alpha,\beta} + g_{0\beta,\alpha} - g_{\alpha\beta,0}\right]$$

nonzero when $\alpha = \beta = 0$, or $\alpha = \beta = 1$

$\alpha = \beta = 0$:

$$\Gamma^0_{00} = \frac{1}{2}\left[\frac{-1}{1+2\phi}\right]\left[g_{00,0}\right]$$

$$= \frac{1}{2}\left[\frac{-1}{1+2\phi}\right]\left[-2\phi_{,0}\right]$$

$$= \frac{\phi_{,0}}{1+2\phi}$$

$$= \phi_{,0}\left[\frac{1}{1-(-2\phi)}\right]$$

$$= \phi_{,0}\left[\sum_{n=0}^{\infty}(-2\phi)^n\right]$$

$$= \phi_{,0}\left[1 - 2\phi + 4\phi^2 - 8\phi^3 + \cdots\right]$$

$$= \phi_{,0} - 2\phi_{,0}\phi + \cdots$$

$$= \boxed{\phi_{,0} + O(\phi^2)}$$

$\alpha = \beta = 1$: $\Gamma^0_{11} = \frac{1}{2}\left[\frac{-1}{1+2\phi}\right]\left[-g_{11,0}\right]$

By prompt, Let g_{11} be $1 + O(\phi)$

$$= \frac{1}{2}\,\frac{-1}{1+2\phi}\,O(\phi_{,0})$$

$$= O(\phi_{,0})\left[1 - 2\phi + 4\phi^2 - \cdots\right]$$

$$= O(\phi_{,0}) + O(\phi^2).$$

This implies $\Gamma^0_{00}\,p^0 p^0$ and $\Gamma^0_{11}\,p^1 p^1$ are both $O(\phi_{,0})$ to leading order. In the nonrelativistic speed limit $p^0 \gg p^1$,

$$\boxed{\underline{\Gamma^0_{00}\,p^0 p^0 \gg \Gamma_{11}\,p^1 p^1}}$$

$\Rightarrow$ g_{11} is irrelevant.

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