Schutz

Schutz. 6.29. metric is $\begin{pmatrix} 1 & \\ & \sin^2\theta \end{pmatrix} \Rightarrow g^{\alpha\beta} = \begin{pmatrix} 1 & \\ & \frac{1}{\sin^2\theta} \end{pmatrix}$.

$$R_{\theta\phi\theta\phi} = \Gamma^{\theta}_{\phi\phi,\theta} - \Gamma^{\theta}_{\phi\theta,\phi} + \Gamma^{\sigma}_{\sigma\theta}\Gamma^{\theta}_{\phi\phi} - \Gamma^{\theta}_{\sigma\phi}\Gamma^{\sigma}_{\phi\theta}.$$

$$= \cancel{\Gamma^{\theta}_{\phi\phi,\theta}} \quad \Gamma^{\gamma}_{\beta\mu} = \frac{1}{2} g^{\alpha\gamma}(g_{\alpha\beta,\mu} + g_{\alpha\mu,\beta} - g_{\beta\mu,\alpha}).$$

$$\Rightarrow \Gamma^{\theta}_{\beta\mu} = \frac{1}{2} g^{\theta\theta}(g_{\theta\beta,\mu} + g_{\theta\mu,\beta} - g_{\beta\mu,\theta}).$$

$$= \frac{1}{2}(g_{\theta\beta,\mu} + g_{\theta\mu,\beta} - g_{\beta\mu,\theta}).$$

$$\Gamma^{\phi}_{\beta\mu} = \frac{1}{2}\frac{1}{\sin^2\theta}(g_{\phi\beta,\mu} + g_{\phi\mu,\beta} - g_{\beta\mu,\phi}).$$

$$\Rightarrow \Gamma^{\theta}_{\phi\phi} = \frac{1}{2}(g_{\theta\phi,\phi} + g_{\theta\phi,\phi} - g_{\phi\phi,\theta}) = -\frac{1}{2} g_{\phi\phi,\theta}$$

$$\Gamma^{\theta}_{\phi\theta} = \frac{1}{2}(g_{\theta\phi,\theta} + g_{\theta\theta,\phi} - g_{\phi\theta,\theta}) = 0.$$

$$\Gamma^{\theta}_{\theta\theta} = \frac{1}{2}(g_{\theta\theta,\theta} + g_{\theta\theta,\theta} - g_{\theta\theta,\theta}) = \frac{1}{2} g_{\theta\theta,\theta} = 0.$$

$$\Gamma^{\phi}_{\phi\theta} = \frac{1}{2}\frac{1}{\sin^2\theta}(g_{\phi\phi,\theta} + g_{\phi\theta,\phi} - g_{\phi\theta,\phi}) = \frac{1}{2}\frac{g_{\phi\phi,\theta}}{\sin^2\theta}$$

$$\Rightarrow R_{\theta\phi\theta\phi} = -\frac{1}{2} g_{\phi\phi,\theta\theta} - \left(-\frac{1}{2} g_{\phi\phi,\theta}\right)\frac{1}{2}\frac{1}{\sin^2\theta} g_{\phi\phi,\theta}$$

$$= -\frac{1}{\cancel{2}}(\cancel{2})(\cos^2\theta - \sin^2\theta) + \cancel{\frac{1}{4}\frac{1}{\sin^2\theta}}\,\cancel{2}^{2}\cos^2\theta\,\cancel{\sin^2\theta}$$

$$= \sin^2\theta - \cos^2\theta + \cos^2\theta = \boxed{\sin^2\theta}$$

All other components of R follow by symmetry.