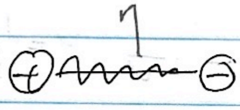
Jackson 4.12 Scratch ①

we want to know natural frequency of molecular oscillation, use classical model



hydrogen.

~~The eq~~ This model valid for any stable equilibria under small oscillations.

$$\cancel{U = \eta^2} \quad F = \frac{-ke^2}{\eta^2} = -\frac{ke^2}{\eta^2}$$

we want a form of $-kx$, or $-m\omega^2\eta$.

expanding. $\frac{1}{\eta^2} = \qquad \vec{F} = -m\omega_0^2\vec{x}$

Consider field inside uniformly charged ball :

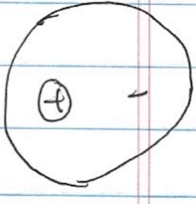
$$\rho = \frac{3e}{4\pi R^3} \qquad \vec{E}(\vec{r}) = \frac{\left(\frac{3e}{4\pi R^3}\frac{4\pi}{3}r^3\right)}{4\pi r^2}$$

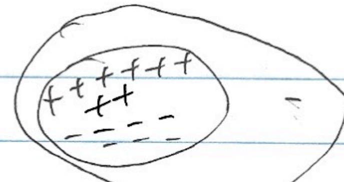
$$= \frac{e}{4\pi}\left(\frac{r}{R}\right)^3\frac{1}{r^2}$$

$$= \frac{e}{4\pi R^3}r.$$

This is already in form of $\vec{F} = kr$, so nice. I was ~~this~~ expecting something ugly then expand to get the harmonic oscillator term.

$\boxed{\text{Jackson 4.12 scratch ②}}$





$$E = \frac{e}{4\pi R^3} r \Rightarrow \cancel{\frac{2}{F}}=$$

$$= \cancel{k} m\omega_0^2 r. \Rightarrow \sqrt{\frac{e_1 e_2}{4\pi R^3 m}} = \cancel{m}\omega_0$$

$$\omega^2 = \frac{1}{t^2}$$

$$\omega_0 = \sqrt{\frac{e_1 e_2}{4\pi R^3 m}} \qquad \frac{e^2}{R^3} \sim \frac{F}{m}$$

~~$m = m_e$.~~ H_2O has $2H$, $1O$.

$$\frac{e^2}{\cancel{4\pi}}$$

H: $m_j = m_e$, $e_j = e$. $\omega_j = \sqrt{\frac{e^2}{4\pi R^3 m}}$. $\frac{F_c^2}{R^2} \sim F$

O: $m_j = m_e$, $e_j = e$ $\omega_j = \sqrt{\frac{e^2}{4\pi R^3 m}}$. $\sim \frac{kg\frac{m}{s^2}}{m^3}$

$$R_H = a_0 = \frac{\hbar^2}{mke^2} = 0.53 \text{ Å} \qquad k = \frac{1}{4\pi\varepsilon_0}$$

$$R_O = \frac{\hbar^2}{mke^2 Z} = \frac{\hbar^2}{mke^2 8} = \frac{0.53}{8} \text{ Å}$$

$$\approx 0.067 \text{ Å}$$

$$\omega_H^2 = \frac{e^2}{4\pi R_H^3 m_e} = \frac{e^2}{4\pi m_e} \frac{m_e^3 k^2 e^6}{\hbar^6}$$

$$= \frac{\varepsilon_0 k e^2}{\cancel{k} m_e} \frac{m_e^2 k^2 e^6}{\hbar^6} = \frac{\varepsilon_0 k^3 e^6 m_e^2}{\hbar^6}$$

$$\frac{e_H^2}{m_H \omega_H^2} = \frac{e^2}{m_e \ldots}$$

$a_0^3 = 0.125 \times 10^{-30}$

$\pi \times 10^{-30}$

$= 10^{-31} \times 10^{-29}$

$\frac{10^{-38}}{10^{-30} \times 10^{-31}} = 10^{22}$

Jackson 4.12 (3)
Scratch

$$\frac{e^2}{m\omega^2} = e \qquad \omega_H^2 = \frac{e^2}{4\pi m a_0^3}$$

$$\frac{e}{m\omega^2} = \frac{\cancel{e^2}}{\cancel{m}} \frac{4\pi \cancel{m} a_0^3}{\cancel{e^2}}$$

$$= 4\pi a_0^3$$

$$\omega_0^2 = \frac{e^2}{4\pi m_e (a_0/8)^2}$$

$$\frac{e^2}{m\omega^2} = \frac{e^2}{m_e \omega^2} = \frac{\cancel{e^2}}{\cancel{m_e}} \frac{4\pi \cancel{m_e} (a_0/8)^3}{\cancel{e^2}}$$

$$= 4\pi \left(\frac{a_0}{8}\right)^3.$$

$$\Rightarrow \gamma_1 \simeq 2\times(4\pi a_0^3) + 1\times\left(4\pi \left(\frac{a_0}{8}\right)^3\right)$$

$$\simeq 4\pi a_0^3 \left[2 + \frac{1}{8^3}\right]$$

$$\simeq 8\pi a_0^3.$$

$$\boxed{\gamma_1 \simeq 3.74\times10^{-30}\ m^3.}$$

$$\simeq$$

$$N = 2.7\times10^{25}\ m^3.$$

$$\Rightarrow \boxed{\gamma_{mol} \simeq 8\pi a_0^3 + \frac{1}{3\epsilon_0}\frac{p_0^2}{kT}}$$

$$\chi_e = \frac{\epsilon}{\epsilon_0} - 1 = \frac{N\gamma_{mol}}{1 - \frac{1}{3}N\gamma_{mol}}.$$

$$(N = 2.7\times10^{25})$$

$$N = 2.7\times10^{25}$$

$$\cancel{}5.6\times10^{23}.$$

V_{3}

~~now we need p_0.~~

Jackson 4.12 scratch

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Ideal Gas law: $PV = nRT$.

$$\chi_e = \frac{N\gamma_{mol}}{1-\frac{1}{3}N\gamma_{mol}} = \frac{1}{\frac{1}{N\gamma_{mol}} - \frac{1}{3}}$$

$$= \left[\frac{1}{N\gamma_{mol}} - \frac{1}{3}\right]^{-1}$$

$$\gamma_{mol} = 8\pi a_0^3 + \frac{1}{3\epsilon_0}\frac{p_0^2}{k}\frac{1}{T} \quad = \left[\frac{1}{N}\frac{1}{8\pi a_0^3 + \frac{1}{3\epsilon_0}\frac{p_0^2}{kT}} - \frac{1}{3}\right]^{-1}$$

$$= a + b\frac{1}{T}$$

$$\Rightarrow \chi_e = \left[\frac{1}{N}\left[\frac{1}{a+b\frac{1}{T}}\right] - \frac{1}{3}\right]^{-1}$$