Polchinski 3.4

Gauge fixed by condition $F^A(\phi) = 0$

$$\int \frac{[d\phi_i]}{V_{gauge}} e^{-S[\phi_i]}$$ gauge fixed by

$$\int \frac{[d\phi_i]}{V_{gauge}} \delta^A[F^A(\phi_i)] e^{-S[\phi_i]}$$

Similar to Polchinski in 3.3.11, we define

$$1 = \Delta(\phi_i) \int [d\zeta] \delta^A[F^A(\phi_i) - F^A(\hat{\phi}_i^\zeta)]$$

where ζ denotes gauge-freedom, $\hat{\phi}_i$ by definition is a fiducial field satisfying $F^A[\hat{\phi}_i] = 0$.

Then insert this 1 we obtain

$$\int \frac{[d\phi_i]}{V_{gauge}} e^{-S[\phi_i]} \Delta(\phi_i) \int [d\zeta] \delta^A[F^A(\phi_i) - F^A(\hat{\phi}_i^\zeta)] \cancel{e^{-S[\phi_i]}}$$

$$= \int \frac{[d\phi_i][d\zeta]}{V_{gauge}} \Delta(\phi_i) \delta^A[F^A(\phi_i) - F^A(\hat{\phi}_i^\zeta)] e^{-S[\phi_i]}$$

Expanding the ~~delta~~ δ-function in fourier modes:

$$= \int \frac{[d\phi_i \, d\zeta \, dB_A]}{V_{gauge}} \Delta(\phi_i) \, e^{iB_A[F^A(\phi_i) - F^A(\hat{\phi}_i^{\zeta})]} \, e^{-S[\phi_i]}$$

$$= \int [d\phi_i] \left\{ \int [dB_A] \left[\int \frac{[d\zeta]}{V_{gauge}} e^{-iB_A F^A(\hat{\phi}_i^{\zeta})} \right] e^{iB_A F^A(\hat{\phi}_i)} \right\} \Delta(\phi_i) \, e^{-S[\phi_i]}$$

$$\Uparrow$$

An integral over functional space ζ, where each ζ is weighted by $e^{-iBF(\zeta(\hat{\phi}_i^{\zeta}))}$, As long as B, F are real, the weight is a pure phase, and $[d\zeta]$ cancels V_{gauge}, so we can write this part as an overall phase dependant only on B, F : $\Gamma(B_A, F^A)$. We expect the B_A dependence ~~to be eliminated once this term is integrated over $[dB_A]$~~, ~~so this term contributes~~ an overall phase only dependent on F^A, the initially chosen gauge, we denote it by $\Gamma(F)$

$$\left(\begin{array}{c}\text{Previous} \\ \text{term}\end{array}\right) = \Gamma(F) \int [d\phi_i][dB_A] \, \Delta(\phi_i) \, e^{iB_A F^A(\hat{\phi}_i)} \, e^{-S[\phi_i]}$$

We now solve for $\Delta^{\phi}(\phi_i)$, we already have my defn.

$$\Delta^{-1}(\phi_i) = \int [d\xi]\ \delta^A[F^A(\phi_i) - F^A(\hat{\phi}_i^{\xi})]$$

We expand $F^A(\hat{\phi}_i^{\xi})$ around ϕ_i over gauge freedoms ε^α:

$$\delta^A[F^A(\phi_i) - F^A(\hat{\phi}_i^{\xi})] = \delta^A[\varepsilon^\alpha \delta_\alpha F^A(\phi_i)]$$

(Recall we have been given gauge transformation $\varepsilon^\alpha \delta_\alpha$)

$$\Rightarrow \Delta^{-1}(\phi_i) = \int [d\varepsilon^\alpha]\ \delta^A[\varepsilon^\alpha \delta_\alpha F^A(\phi_i)]$$

using integral rep. of $\delta^A(\cdots)$ $\Rightarrow$

$$= \int [d\varepsilon^\alpha d\beta_A] \exp\left\{-2\pi i \int d^d\sigma\ \beta_A \varepsilon^\alpha \delta_\alpha F^A(\phi)\right\}$$

Recall that for (x,y) bosonic, (ψ,χ) fermionic, we have a way to invert something like this.

$$\int_{-\infty}^{\infty} dx \int_{-\infty}^{\infty} dy\ \exp(2\pi i \lambda x y) = \left[\int d\psi \int d\chi\ \exp(\lambda \chi \psi)\right]^{-1} \quad (A.2.2?)$$

$$\Rightarrow \Delta(\phi_i) = \int [dc^\alpha db_A]\ e^{-b_A c^\alpha \delta_\alpha F^A(\phi)}$$

for fermionic fields c^α, b_A,

Putting everything together, we have.

$$\int \frac{[d\phi_i]}{V_{gauge}} e^{-S[\phi]}$$

$$= \Gamma(F) \int [d\phi_i \, dB_A \, db_A \, dc^\alpha] \, e^{-S[\phi]} \, e^{iB_A F^A(\phi)} \, e^{-b_A c^\alpha \delta_\alpha F^A(\phi)}$$

$$= \Gamma(F) \int [d\phi_i dB_A \, db_A \, dc^\alpha] \, e^{-S_1 - S_2 - S_3}$$

$$S_1 = S[\phi],$$
$$S_2 = -B_A F^A(\phi)$$
$$S_3 = b_A c^\alpha \delta_\alpha F^A(\phi).$$

This gauge-dependent phase should cancel when computing expectation values, it shows up here because we are working with the generating functional / partition function $Z[\phi]$.

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