Polchinski 2.6

Write $V_b = V_b(\sigma(x^a))$, then we have constraint

$$\frac{\partial V_b}{\partial \sigma}\frac{\partial \sigma}{\partial x^a} + \frac{\partial V_a}{\partial \sigma}\frac{\partial \sigma}{\partial x^b} = \omega\, g_{ab}.$$

- Fix $\omega = 1$, this removes 1 Dof. $\qquad D = 1$
- Fix $\frac{\partial V_b}{\partial \sigma} = 1$, this removes d Dof $\qquad D = 1 + d$

After applying the 2 constraints above,

we now have

$$\frac{\partial \sigma}{\partial x^a} + \frac{\partial \sigma}{\partial x^b} = g_{ab} = \delta_{ab}$$

- For $a = b$, this is fixed completely:

$$\frac{\partial \sigma}{\partial x^a} = \frac{1}{2}, \quad \text{this}$$

has no Dof.

- For $a \neq b$, this has $\frac{d(d-1)}{2}$ Dof, corresponding to # of independent components of a $d \times d$ antisymmetric tensor $\qquad D = 1 + d + \frac{d(d-1)}{2}.$

- Are we done? No, each V_b still admits an arbitrary constant: $V_b = V_b(\sigma) + \beta$ $\qquad D = 1 + d + \frac{d(d-1)}{2} + d.$

So we have in total, $1 + 2d + \frac{d(d-1)}{2} = \frac{2 + 4d + d^2 - d}{2}$

$$= \boxed{\frac{(d+1)(d+2)}{2}} \text{ Dof}$$

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