

## Polchinski 2.3 (a)

$$\text{For } n=2, \prod_{i=1}^n e^{-ik_i \cdot X(z_i, \bar{z}_i)} = e^{ik_1 X_1} \dots e^{ik_2 X_2}$$

where we have used  $X_1$  to denote  $X(z_1, \bar{z}_1)$ .

Then the exercise states

$$\langle e^{ik_1 X_1} \dots e^{ik_2 X_2} \rangle = i C^X (2\pi)^D f^D(k_1 + k_2) |z_1 - z_2|^{\alpha' k_1 k_2}$$

Recall (eq. 2.2.14) states

$$e^{ik_1 \cdot X(z, \bar{z})} e^{ik_2 \cdot X(0,0)} = |z|^{\alpha' k_1 k_2} e^{i(k_1 + k_2) \cdot X(0,0)} [1 + O(z, \bar{z})]$$

Apply  $\langle \dots \rangle$  gives

$$\langle e^{ik_1 X(z)} e^{ik_2 X(0)} \rangle = |z|^{\alpha' k_1 k_2} \langle e^{i(k_1 + k_2) X(0)} [1 + O(z, \bar{z})] \rangle$$

To zeroth-order, this is

$$|z|^{\alpha' k_1 k_2} \langle e^{i(k_1 + k_2) X(0)} \rangle$$

$$= |z|^{\alpha' k_1 k_2} (2\pi)^D f^D(k_1 + k_2)$$

The factor of  $(2\pi)^D$  comes from the fact that the  $\delta$ -function is on variables in phase space.