Polchinski 2.1(a)

By defn, $z = a + ib$, $\bar{z} = a - ib$

$$\partial_z = \frac{1}{2}[\partial_a - i\partial_b], \quad \partial_{\bar{z}} = \frac{1}{2}[\partial_a + i\partial_b]$$

* $$\bar{\partial} \ln|z|^2 = \partial_{\bar{z}} \ln|a^2+b^2|$$

$$= \frac{1}{2}\left[\frac{2a}{a^2+b^2} - i\frac{(2b)}{a^2+b^2}\right]$$

$$= \frac{a-ib}{a^2+b^2} = \boxed{\frac{1}{z}}$$

$$\Rightarrow \partial\bar{\partial} \ln|z|^2 = \partial \frac{1}{z}$$

$$= \bar{\partial}\frac{1}{z}$$

* $$\int_R d^2z\,(\partial_z v^z + \partial_{\bar{z}} v^{\bar{z}}) = i\oint_{\partial R} (v^z d\bar{z} - v^{\bar{z}} dz)$$

Let $v = \frac{1}{z}$, then

$$\int_R d^2z\,(\partial\bar{\partial}\ln|z|^2) = \int_R d^2z\left[\partial\frac{1}{z}\right]$$

$$= \int_R d^2z\left[\partial\frac{1}{z} + \bar{\partial}\frac{1}{z}\right]$$

$$= i\oint_{\partial R}\left[-\frac{1}{z}dz\right]$$

$$= -i[2\pi i] = \boxed{2\pi}$$

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