Polchinski 2.9 (a)

linear dilaton | Define $T_L(z) = \frac{-1}{\alpha'} :\partial X^\mu \partial X_\mu: + V_\mu \partial^2 X^\mu$

$$= T(z) + T'(z)$$

$T(z)$ is the T in section 2.4, $T_L(z)$ is the T in linear dilaton theory.

Then

$$T_L(z) T_L(0) = T(z) T(0) + T(z) T'(0) + T'(z) T(0) + T'(z) T'(0)$$

$T = :\partial X^\mu \partial X_\mu:$ is nicely behaving because it's normal ordered.
$\partial^2 X$ is also nicely behaving because it's only 1 field.

~~Test~~ So we expect TT' and $T'T$ to be nicely behaving, yet $T'T' = \partial^2 X \partial^2 X$ is not. Computing it explicitly:

$$T'(z) T'(0) = V_\mu \partial^2 X^\mu V_\nu \partial^2 X^\nu$$

$$= V_\mu V_\nu \partial^2 X^\mu \partial^2 X^\nu$$

$$= V_\mu V_\nu \left[:\partial^2 X^\mu \partial^2 X^\nu: - \eta^{\mu\nu} \frac{\alpha'}{2} \partial^2 \frac{1}{z^2} \right]$$

$$\sim -V_\mu V^\mu \frac{\alpha'}{2} \frac{6}{z^4}$$

$$\Rightarrow T'(z)T'(0) \sim \frac{-D}{2z^4} - \frac{6\alpha' V_\mu V^\mu}{2z^4} + O\left(\frac{1}{z^3}\right)$$

$$\Rightarrow c = D + 6\alpha' V_\mu V^\mu$$

Polchinski 2.9

b, c theory we are given $:b(z_1)\,c(z_2): = b(z_1)\,c(z_2) - \frac{1}{z_{12}}$,

this is a consequence of $b_1 c_2 \sim \frac{1}{z_{12}}$.

$T(z) = :(\partial b)c: - \lambda\,\partial :bc:$,

we consider $T(z)\,T(0)$

$$T(z)\,T(0) = \left\{ :(\partial b)c|_z: - \lambda\,\partial :bc|_z: \right\} \left\{ :(\partial b)c|_0: - \lambda\,\partial :bc|_0: \right\}$$

This product will contain 3 kinds of terms:

1. $:(\partial b)c|_z: \, :(\partial b)c|_0:$

2. $:(\partial b)c|_z: \partial :bc|_0:$

3. $\partial :bc|_z: \partial :bc|_0:$

Type 1 is easy to work out:

$$:(\partial b)c|_z: \, :(\partial b)c|_0:$$

$$= :(\partial b)c|_z (\partial b)c|_0: - \partial\frac{1}{z} :c|_z (\partial b)|_0:$$

$$- \partial\frac{1}{z} :(\partial b)|_z\, c|_0:$$

$$- \partial\frac{1}{z}\, \partial\frac{1}{z}$$

$$= \,:(\partial b)c|_z (\partial b)c|_0: \; - \; \partial\frac{1}{z} :(c|_0 + z\,\partial c|_0)\,\partial b|_0 :$$

$$- \; \partial\frac{1}{z} :(\partial b|_0 + z\,\partial^2 b|_0)\,\cancel{\partial} c|_0 :$$

$$- \; \frac{1}{z^4}$$

The component that will contribute to the charge will be the non-field term, which is $-\frac{1}{z^4}$

Now we evaluate type 2.

$$:(\partial b)c|_z : \partial :bc|_0:$$

$$\partial :bc: \; = \; \partial\left[bc - \frac{1}{z} \right]$$

$$= (\partial b)c + b(\partial c) - \partial\frac{1}{z}$$

$$= :(\partial b)c: + :b(\partial c): + \partial\frac{1}{z}.$$

$$\Rightarrow \; :(\partial b)c|_z : \partial :bc|_0:$$

$$= \; :(\partial b)c|_z : \left[:(\partial b)c|_0: + :b(\partial c)|_0: + \partial\frac{1}{z}\Big|_0 \right]$$

$$= \; :\cdots: \; - \left(\partial\frac{1}{z}\right)\left(\partial\frac{1}{z}\right) - \left(\partial^2\frac{1}{z}\right)\frac{1}{z} + \partial\frac{1}{z} \times (\text{field terms}).$$

$\Rightarrow$ The contributions to charge will be $-\frac{3}{z^4}$

Lastly, we evaluate type 3:

$$\partial :bc|_z : \partial : bc|_0$$

$$= \left[:(\partial b)c: + :b(\partial c): + \partial\frac{1}{z}\right]_z \left[:(\partial b)c: + :b(\partial c): + \partial\frac{1}{z}\right]_0$$

$$\sim -\left[\partial\frac{1}{z}\,\partial\frac{1}{z} + \left(\partial^2\frac{1}{z}\right)\frac{1}{z} + \partial\frac{1}{z}\,\partial\frac{1}{z} + \left(\partial^2\frac{1}{z}\right)\frac{1}{z} + \text{Field terms}\right]$$

$$\sim -\frac{6}{z^4}$$

again, we ignored field terms that won't contribute to the central charge.

$\Rightarrow$ The contribution from $\frac{1}{z^4}$ terms in $T(z)T(0)$ will look like

$$-\frac{1}{z^4} - 2\lambda\left(-\frac{3}{z^4}\right) + \lambda^2\left(-\frac{6}{z^4}\right)$$

$$= \frac{-6\lambda^2 + 6\lambda^2 - 1}{z^4}$$

$$= \frac{-12\lambda^2 + 12\lambda^2 - 1}{2z^4}$$

$$= \boxed{\frac{(-3)(2\lambda - 1)^2 + 1}{2z^4}}$$

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β, γ theory ~~We take~~ In this theory, we now have

$$\beta_1 \gamma_2 \sim \frac{-1}{z_{12}}, \text{ implying } :\beta_1 \gamma_2: = \beta\gamma + \frac{1}{z_{12}}$$

This inverts the sign of cross-contraction in all $:\mathcal{F}::G:$ type calculations we computed in the b, c theory, so will flip all charge-contributing terms we found in the b, c theory (Recall all charge-contributing terms in b, c theory came from cross-contraction).

$$\Rightarrow \quad c = (3)(2\lambda - 1)^2 + 1$$

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