Polchinski 1.2

Show that the classical EQM implies that the ends of the open string moves at speed of light.

$$S_{\text{Polyakov}} = \int_{\mathcal{M}} d\tau\, d\sigma\, \mathcal{L}_{\text{Polyakov}}$$

$$\mathcal{L}_{\text{Polyakov}} = (-\gamma)^{1/2} \gamma^{ab} \partial_a X^\mu \partial_b X_\mu$$

EQM: $$0 = \partial_a \left[(-\gamma)^{1/2} \gamma^{ab} \partial_b X_\mu \right]$$

$$\partial_a \left[(-\gamma)^{1/2} \partial^a X_\mu \right] = 0$$

$$\frac{1}{2} (-\gamma)^{-1/2} (\partial_a \gamma) \partial^a X_\mu + (-\gamma)^{1/2} \partial_a \partial^a X_\mu = 0$$

Simplify:

$$\frac{1}{2} (-\gamma)^{-1} (\partial_a \gamma) \partial^a X_\mu + \partial_a \partial^a X_\mu = 0$$

The boundary of open string satisfies Neumann boundary condition

$$n^a \partial_a X_\mu = 0 \quad \text{for } n^a \text{ normal to } \partial\mathcal{M}.$$

γ is a scalar, $\partial_a \gamma$ is the 1-form from this scalar, so it is the normal surface.

$\Rightarrow$ EQM simplifies $\partial_a \partial^a X_\mu = 0$.

$$\boxed{(\partial_\sigma^2 - \partial_\tau^2) X_\mu = 0}$$

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6.23.2024
HBD to my Dad!