Schwartz 16.1

$$\Pi^{\mu\nu} = ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{-4k^\mu k^\nu + 2p^\mu k^\nu + 2p^\nu k^\mu - p^\mu p^\nu + 2g^{\mu\nu}[(p-k)^2 - m^2]}{[(k-p(1-x))^2 + p^2x(1-x) - m^2 + i\varepsilon]^2}$$

$$k^\mu \to k^\mu + p^\mu(1-x)$$

$$= ie^2 \int \frac{d^4k}{(2\pi)^4} \frac{\begin{array}{l} -4[k^\mu + p^\mu(1-x)][k^\nu + p^\nu(1-x)] + 2p^\mu[k^\nu + p^\nu(1-x)] \\ + 2p^\nu[k^\mu + p^\mu(1-x)] \\ - p^\mu p^\nu \\ + 2g^{\mu\nu}[(xp-k)^2 - m^2] \end{array}}{[k^2 + p^2x(1-x) - m^2 + i\varepsilon]^2}$$

Examine numerator:

$$-4[k^\mu k^\nu + p^\mu k^\nu(1-x) + k^\mu p^\nu(1-x) + p^\mu p^\nu(1-x)]$$
$$+ 2p^\mu k^\nu + 2p^\mu p^\nu(1-x) + 2p^\nu k^\mu + 2p^\nu p^\mu(1-x) - p^\mu p^\nu$$
$$+ 2g^{\mu\nu}[x^2p^2 + k^2 - 2xp\cdot k - m^2]$$

($p\cdot k$ drops by antisymmetry)

$$= -4k^\mu k^\nu - 4p^\mu k^\nu(1-x) - 4k^\mu p^\nu(1-x) - 4p^\mu p^\nu(1-x)^2$$
$$+ 2p^\mu k^\nu \quad + 2k^\mu p^\nu \quad + 2p^\mu p^\nu(1-x)$$
$$+ 2p^\mu p^\nu(1-x)$$
$$- p^\mu p^\nu$$
$$+ 2g^{\mu\nu}[x^2p^2 + k^2 - m^2]$$

$$= -4k^\mu k^\nu - p^\mu k^\nu(2-4x) - k^\mu p^\nu(2-4x) - p^\mu p^\nu[4x^2 - 4x + 1]$$
$$+ 2g^{\mu\nu}[x^2p^2 + k^2 - m^2]$$

$p^\mu k^\nu$ will vanish bc it's antisymmetric about $k^\nu = 0$.

So we write the integral as

$$2ie^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{-2k^\mu k^\nu - p^\mu p^\nu [2x^2-2x+\frac{1}{2}] + g^{\mu\nu}[x^2p^2+k^2-m^2]}{[k^2+p^2x(1-x)-m^2+i\varepsilon]^2}$$

Let $\Delta = m^2 - p^2 x(1-x)$, (Schwartz 16.33)

use dimensional regularization : $k^\mu k^\nu \to \frac{1}{d} k^2 g^{\mu\nu}$

$$2ie^2 \int_0 dx \int \frac{d^dk}{(2\pi)^d} \frac{-\frac{2}{d}k^2 g^{\mu\nu} + g^{\mu\nu}[x^2p^2+k^2-m^2] - p^\mu p^\nu [2x^2-2x+\frac{1}{2}]}{[k^2-\Delta+i\varepsilon]^2}$$

Do the two integrals accordingly (Schwartz 16.34, 16.35)

$$2ie^2 \int_0 dx \Big\{ g^{\mu\nu}\Big[1-\frac{d}{2}\Big]\Big[\frac{i}{(4\pi)^{d/2}}\Big(\frac{1}{\Delta}\Big)^{1-\frac{d}{2}} \Gamma\Big(1-\frac{d}{2}\Big)$$

$$+ \Big[g^{\mu\nu}(x^2p^2-m^2) - p^\mu p^\nu \Big[2x^2-2x+\frac{1}{2}\Big]\Big] \frac{i}{(4\pi)^{d/2}}\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}} \Gamma\Big(2-\frac{d}{2}$$

$$= \frac{2ie^2 \, i}{(4\pi)^{d/2}} \Gamma\Big(2-\frac{d}{2}\Big) \int_0^1 dx \Big\{ g^{\mu\nu}\Big(\frac{1}{\Delta}\Big)^{1-d/2}$$

$$+ \Big[g^{\mu\nu}(x^2p^2-m^2) - p^\mu p^\nu \Big(2x^2-2x+\frac{1}{2}\Big)\Big]\Big(\frac{1}{\Delta}\Big)^{2-\frac{d}{2}}$$

$$= \frac{-2e^2}{(4\pi)^{d/2}} \Gamma\Big(2-\frac{d}{2}\Big) \int_0^1 dx \Big\{ g^{\mu\nu} p^2 x(2x-1)$$

$$- p^\mu p^\nu \Big(x-\frac{1}{2}\Big)(2x-1) \Big\} \Big(\frac{1}{\Delta}\Big)^{2-d/2}$$

$$\int_0^1 dx \left(\frac{1}{2}\right)(2x-1) = \int_0^1 \left(x-\frac{1}{2}\right) dx$$

Notice the antisymmetry.

$$\Rightarrow \Pi^{\mu\nu} = \frac{-2e^2}{(4\pi)^{d/2}} \Gamma\left(2-\frac{d}{2}\right) \int_0^1 dx \left\{ g^{\mu\nu} p^2 x(2x-1) - p^\mu p^\nu x(2x-1) \right\} \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}}$$

$$= \frac{-2e^2}{(4\pi)^{d/2}} \Gamma\left(2-\frac{d}{2}\right) \left[g^{\mu\nu} p^2 - p^\mu p^\nu\right] \int_0^1 dx\, x(2x-1) \left(\frac{1}{\Delta}\right)^{2-\frac{d}{2}}$$

This matches (16.38).

$p_\mu \left[g^{\mu\nu} p^2 - p^\mu p^\nu\right] = 0$, so the Ward Identity is satisfied.

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