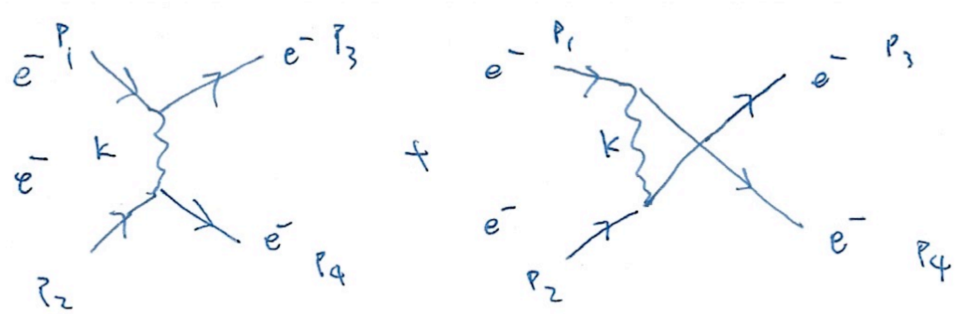

Schwartz 13.1 pg 1

$$i\mathcal{M} = \bar{u}(p_3)(+ie\gamma^\mu)u(p_1)\left[\frac{-ig_{\mu\nu}}{k^2+i\varepsilon}\right]\bar{u}(p_4)(+ie\gamma^\nu)u(p_2)$$

$$+\bar{u}(p_4)(+ie\gamma^\mu)u(p_1)\left[\frac{-ig_{\mu\nu}}{k^2+i\varepsilon}\right]\bar{u}(p_3)(+ie\gamma^\nu)u(p_2)$$

$$i\mathcal{M} = i\frac{e^2}{k^2}\Big\{[\bar{u}_3\gamma^\mu u_1][\bar{u}_4\gamma_\mu u_2] + [\bar{u}_4\gamma^\mu u_1][\bar{u}_3\gamma_\mu u_2]\Big\}$$

where we have used $\bar{u}_3$ to denote $\bar{u}(p_3)$, etc.

$$\Rightarrow \mathcal{M} = \frac{e^2}{s}\Big\{[\bar{u}_3\gamma^\mu u_1][\bar{u}_4\gamma_\mu u_2] + [\bar{u}_4\gamma^\mu u_1][\bar{u}_3\gamma_\mu u_2]\Big\}$$

where $s = k = (p_1+p_2)^2$

Then it's straightforward to compute $|M|^2$

$|M|^3 = M^\dagger M$

$$= \frac{e^4}{s^2}\left\{ [u_2^\dagger \gamma_0 \gamma_\mu^\dagger \gamma_0 u_4][u_1^\dagger \gamma_0 \gamma^{\mu\dagger} \gamma_0 u_3][\bar{u}_3 \gamma^\nu u_1][\bar{u}_4 \gamma_\nu u_2] + \vdots \right\}$$

$$= \frac{e^4}{s^2}\left\{ [\bar{u}_2 \gamma_\mu u_4][\bar{u}_1 \gamma^\mu u_3][\bar{u}_3 \gamma^\nu u_1][\bar{u}_4 \gamma_\nu u_2] + \vdots \right\}$$

$$= \frac{e^4}{s^2}\left\{ [\bar{u}_2 \gamma_\mu u_4][\bar{u}_4 \gamma_\nu u_2][\bar{u}_1 \gamma^\mu u_3][\bar{u}_3 \gamma^\nu u_1] + \vdots \right\}$$

$$= \frac{e^4}{s^2}\left\{ \begin{aligned} &[\bar{u}_2 \gamma_\mu u_4][\bar{u}_4 \gamma_\nu u_2][\bar{u}_1 \gamma^\mu u_3][\bar{u}_3 \gamma^\nu u_1] + \\ &[\bar{u}_2 \gamma_\mu u_4][\bar{u}_4 \gamma^\nu u_1][\bar{u}_1 \gamma^\mu u_3][\bar{u}_3 \gamma_\nu u_2] + \\ &[\bar{u}_2 \gamma_\mu u_3][\bar{u}_3 \gamma^\nu u_1][\bar{u}_1 \gamma^\mu u_4][\bar{u}_4 \gamma_\nu u_2] + \\ &[\bar{u}_2 \gamma_\mu u_3][\bar{u}_3 \gamma_\nu u_2][\bar{u}_1 \gamma^\mu u_4][\bar{u}_4 \gamma^\nu u_1] \end{aligned} \right\}$$

we will sum over all spin states, then attach a factor of $\frac{1}{4}$ for average of initial states.

Let the sum over spin states be implicit,

$$\bar{u}_2 \gamma_\mu u_4 \bar{u}_4 \gamma_\nu u_2 = \bar{u}_2 \gamma_\mu [\not{p}_4 + m \mathbb{1}] \gamma_\nu u_2$$

$$= \mathrm{Tr}[(\not{p}_4 + m)\gamma_\mu (\not{p}_2 + m)\gamma_\nu]$$

$$= \mathrm{Tr}[\gamma_\alpha \gamma_\mu \gamma_\beta \gamma_\nu p_4^\alpha p_2^\beta + m^2 \gamma_\mu \gamma_\nu]$$

$$= p_4^\alpha p_2^\beta \, 4[g_{\alpha\mu} g_{\beta\nu} - g_{\alpha\beta} g_{\mu\nu} + g_{\alpha\nu} g_{\mu\beta}] + 4m^2 g_{\mu\nu}$$

$$= 4[p_{4\mu} p_{2\nu} - p_4 \cdot p_2 \, g_{\mu\nu} + p_{4\nu} p_{2\mu}] + 4m^2 g_{\mu\nu}$$

$$[\bar{u}_2 \gamma_\mu u_4 \bar{u}_4 \gamma_\nu u_2][\bar{u}_1 \gamma^\mu u_3][\bar{u}_3 \gamma^\nu u_1]$$

$$= 4[p_{4\mu} p_{2\nu} + p_{2\mu} p_{4\nu} + (m^2 - p_2 \cdot p_4) g_{\mu\nu}] \, 4[p_3^\mu p_1^\nu + p_1^\mu p_3^\nu + (m^2 - p_1 \cdot p_3) g^{\mu\nu}]$$

$$\begin{aligned} = 16[\, & p_{12} p_{34} + p_{23} p_{14} + (m^2 - p_{24}) p_{13} \\ & + p_{14} p_{23} + p_{12} p_{34} + (m^2 - p_{24}) p_{13} \\ & + p_{12}(m^2 - p_{13}) + p_{24}(m^2 - p_{13}) + 4(m^2 - p_{24})(m^2 - p_{13}) \,] \end{aligned}$$

$\boxed{\text{unfinished.}}$

Davidson Chang

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