Schwartz

10.1 (a)

Dirac Eq: $(i\gamma^\mu \partial_\mu - m)\psi = 0.$

$$i\gamma^0 \partial_0 \psi \cancel{+} - i\vec{\gamma}\cdot\vec{\nabla}\psi - m\psi = 0.$$

$H_D \psi = i\partial_t \psi$, we write Dirac eq as

$$i\gamma^0 \partial_0 \psi = i\vec{\gamma}\cdot\vec{\nabla}\psi + m\psi$$

$(\gamma^0)^{-1} = \gamma^0$, so

$$i\partial_t \psi = \gamma^0 [i\vec{\gamma}\cdot\vec{\nabla} + m]\psi$$

$\vec{p} = -i\vec{\nabla}$, so

$$i\partial_t \psi = \gamma^0 [-\vec{p}\cdot\vec{\gamma} + m]\psi$$

$$\boxed{H_D = \gamma^0 [-\vec{p}\cdot\vec{\gamma} + m]}$$

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Schwartz
10.1(b)

$$H_0 = \gamma^0 [-\vec{p}\cdot\vec{\gamma} + m]$$

$$\vec{p}\cdot\vec{\gamma} = \vec{p}\cdot\begin{bmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & \vec{p}\cdot\vec{\sigma} \\ -\vec{p}\cdot\vec{\sigma} & 0 \end{bmatrix}$$ Recall $\vec{\gamma} = \begin{pmatrix} 0 & \vec{\sigma} \\ -\vec{\sigma} & 0 \end{pmatrix}$

$$\Rightarrow H_0 = \gamma^0 \begin{bmatrix} m & -\vec{p}\cdot\vec{\sigma} \\ \vec{p}\cdot\vec{\sigma} & m \end{bmatrix}$$ Recall $\gamma^0 = \begin{pmatrix} & 1 \\ 1 & \end{pmatrix}$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} m & -\vec{p}\cdot\vec{\sigma} \\ \vec{p}\cdot\vec{\sigma} & m \end{pmatrix}$$

$$= \begin{bmatrix} \vec{p}\cdot\vec{\sigma} & m \\ m & -\vec{p}\cdot\vec{\sigma} \end{bmatrix}$$

$$H_0 - eA_0 = \begin{bmatrix} \vec{p}\cdot\vec{\sigma} - eA_0 & m \\ m & -\vec{p}\cdot\vec{\sigma} - eA_0 \end{bmatrix}$$

Let $\vec{p}\cdot\vec{\sigma} = a, \quad eA_0 = b,$ then

$$H_0 - eA_0 = \begin{bmatrix} a-b & m \\ m & -a-b \end{bmatrix}$$

$$[H_0 - eA_0]^2 = \begin{bmatrix} a-b & m \\ m & -a-b \end{bmatrix}\begin{bmatrix} a-b & m \\ m & -a-b \end{bmatrix}$$

$$= \begin{bmatrix} (a-b)^2 + m^2 & m(a-b) - m(a+b) \\ m(a-b) - m(a+b) & (a-b)^2 + m^2 \end{bmatrix}$$

$$= \begin{bmatrix} (a-b)^2 + m^2 & -2mb \\ -2mb & (a-b)^2 + m^2 \end{bmatrix}$$

$$(a-b)^2 = (\vec{p}\cdot\vec{\sigma} - eA_0)^2 = (\vec{p}\cdot\vec{\sigma})^2 + e^2A_0^2 - 2eA_0\vec{p}\cdot\vec{\sigma}$$

$$\vec{p}\cdot\vec{\sigma} = \begin{pmatrix} p_3 & p_1 - ip_2 \\ p_1 + ip_2 & -p_3 \end{pmatrix}, \quad (\vec{p}\cdot\vec{\sigma})^2 = \begin{pmatrix} p_3 + p_1 + p_2 & 0 \\ 0 & p_3 + p_1 + p_2 \end{pmatrix}$$

$$= |\vec{p}|^2 \mathbb{I}$$

$$\Rightarrow [H_0 - eA_0]^2 = \begin{bmatrix} |\vec{p}|^2 + m^2 & -2meA_0 \\ -2meA_0 & |\vec{p}|^2 + m^2 \end{bmatrix}$$

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Dimensional analysis: $H_D \sim J \Rightarrow eA_0 \sim J$,

if e is dimensionless, then $A_0 \sim J$.

$(H_D - eA_0)^2$ should have units of J^2

$$[H_D - eA_0]^2 = \begin{bmatrix} |\vec{p}|^2 + m^2 & -2meA_0 \\ -2meA_0 & |\vec{p}|^2 + m^2 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} |\vec{p}|^2c^2 + m^2c^4 & -2mc^2eA_0 \\ -2mc^2eA_0 & |\vec{p}|^2c^2 + m^2c^4 \end{bmatrix}$$