Griffiths

4.28 (a) $\mathcal{M}_a = \langle \pi^+ p | \pi^+ p \rangle$

$$= \left(\langle 1\,1 | \langle \tfrac{1}{2}\,\tfrac{1}{2} | \right) \left(|1\,1\rangle |\tfrac{1}{2}\,\tfrac{1}{2}\rangle \right)$$

$$= \langle \tfrac{3}{2}\,\tfrac{3}{2} | \tfrac{3}{2}\,\tfrac{3}{2} \rangle = \mathcal{M}_3.$$

$\mathcal{M}_b \Leftrightarrow (\pi^0 + p \to \pi^0 + p)$

$$= \tfrac{2}{3} \langle \tfrac{3}{2}\,\tfrac{1}{2} | \tfrac{3}{2}\,\tfrac{1}{2} \rangle + \tfrac{1}{3} \langle \tfrac{1}{2}\,\tfrac{1}{2} | \tfrac{1}{2}\,\tfrac{1}{2} \rangle$$

$$= \tfrac{2}{3} \mathcal{M}_3 + \tfrac{1}{3} \mathcal{M}_1.$$

$\mathcal{M}_c \Leftrightarrow \pi^- + p \to \pi^- + p$

$$= \tfrac{1}{3} \langle \tfrac{3}{2}\, -\tfrac{1}{2} | \tfrac{3}{2}\, -\tfrac{1}{2} \rangle + \tfrac{2}{3} \langle \tfrac{1}{2}\, -\tfrac{1}{2} | \tfrac{1}{2}\, -\tfrac{1}{2} \rangle$$

$$= \tfrac{1}{3} \mathcal{M}_3 + \tfrac{2}{3} \mathcal{M}_1.$$

$\mathcal{M}_d \Leftrightarrow \pi^+ + n \to \pi^+ + n$

$$= \tfrac{1}{3} \langle \tfrac{3}{2}\,\tfrac{1}{2} | \tfrac{3}{2}\,\tfrac{1}{2} \rangle + \tfrac{2}{3} \langle \tfrac{1}{2}\,\tfrac{1}{2} | \tfrac{1}{2}\,\tfrac{1}{2} \rangle$$

$$= \tfrac{1}{3} \mathcal{M}_3 + \tfrac{2}{3} \mathcal{M}_1.$$

$$\mathcal{M}_e = \tfrac{2}{3} \langle \tfrac{3}{2}\, -\tfrac{1}{2} | \tfrac{3}{2}\, -\tfrac{1}{2} \rangle + \tfrac{1}{3} \langle \tfrac{1}{2}\, -\tfrac{1}{2} | \tfrac{1}{2}\, -\tfrac{1}{2} \rangle$$

$$= \tfrac{2}{3} \mathcal{M}_3 + \tfrac{1}{3} \mathcal{M}_1.$$

$\mathcal{M}_f = \mathcal{M}_3.$

$$\mu_g = \frac{1}{\sqrt{3}}\sqrt{\frac{2}{3}}\,\mu_3 - \frac{1}{\sqrt{3}}\sqrt{\frac{2}{3}}\,\mu_1 = \frac{\sqrt{2}}{3}\left[\mu_3 - \mu_1\right]$$

$$\mu_h = \sqrt{\frac{2}{3}}\sqrt{\frac{1}{3}}\left\langle \tfrac{3}{2}\,\tfrac{1}{2}\middle|\tfrac{3}{2}\,\tfrac{1}{2}\right\rangle - \frac{1}{\sqrt{3}}\sqrt{\frac{2}{3}}\left\langle \tfrac{1}{2}\,\tfrac{1}{2}\middle|\tfrac{1}{2}\,\tfrac{1}{2}\right\rangle$$
$$= \frac{\sqrt{2}}{3}\left[\mu_3 - \mu_1\right]$$

$$\mu_i = \frac{1}{\sqrt{3}}\sqrt{\frac{2}{3}}\left\langle \tfrac{3}{2}\,-\tfrac{1}{2}\middle|\tfrac{3}{2}\,-\tfrac{1}{2}\right\rangle - \sqrt{\frac{2}{3}}\frac{1}{\sqrt{3}}\left\langle \tfrac{1}{2}\,-\tfrac{1}{2}\middle|\tfrac{1}{2}\,-\tfrac{1}{2}\right\rangle$$
$$= \frac{\sqrt{2}}{3}\left[\mu_3 - \mu_1\right]$$

$$\mu_j = \frac{\sqrt{2}}{3}\left[\mu_3 - \mu_1\right].$$

(b). $\sigma_g \doteq \sigma_h \doteq \sigma_i \doteq \sigma_j$

$$\sigma_a : \sigma_b : \sigma_c : \sigma_d : \sigma_e : \sigma_f = 9|\mu_3|^2 : |2\mu_3+\mu_1|^2 : |\mu_3+2\mu_1|^2$$
$$: |\mu_3+2\mu_1|^2 : |2\mu_3+\mu_1|^2 : 9|\mu_3|^2.$$

Or more succinctly,

$$\sigma_b = \sigma_e, \quad \sigma_a = \sigma_f, \quad \sigma_c = \sigma_d, \quad \sigma_g = \sigma_h = \sigma_i = \sigma_j$$

$$\boxed{\sigma_b : \sigma_a : \sigma_c : \sigma_g = \cancel{9|\mu_3|^2 :}\ |2\mu_3+\mu_1|^2 : 9|\mu_3|^2 : |\mu_3+2\mu_1|^2 : 2|\mu_3-\mu_1|^2}$$

(d) In $\mu_3 \gg \mu_1$ approx, we have

$$\boxed{\sigma_b : \sigma_a : \sigma_c : \sigma_g \simeq 4 : 9 : 1 : 2}$$