Griffiths

4.27. $(I_1 + I_2)^2 - I_1^2 - I_2^2 = 2\vec{I}_1 \cdot \vec{I}_2$

$$\Rightarrow \vec{I}_1 \cdot \vec{I}_2 = \frac{1}{2}\left[(\vec{I}_1 + \vec{I}_2)^2 - I_1^2 - I_2^2\right]$$

$$= \frac{1}{2}\left[(\vec{I}_1 + \vec{I}_2)^2 - \left(\cancel{\frac{3}{4}} + \cancel{\frac{3}{4}}\ \frac{3}{2}\right)\right]$$

$$= \frac{1}{2}(\vec{I}_1 + \vec{I}_2)^2 - \frac{1}{2}\left(\frac{3}{2}\right)$$

$$= \frac{1}{2}(\vec{I}_1 + \vec{I}_2)^2 - \frac{3}{4}.$$

Suppose $\vec{I}_1 + \vec{I}_2$ is a triplet, then $I_{tot} = 1$,

$$\frac{1}{2}(\vec{I}_1 + \vec{I}_2)^2 = \frac{1}{2}(1)(1+1) = 1.$$

then $\vec{I}_1 \cdot \vec{I}_2 = 1 - \frac{3}{4} = \boxed{\frac{1}{4}}$

Suppose $\vec{I}_1 + \vec{I}_2$ is a singlet, then $I_{tot} = 0$,

$$\frac{1}{2}(\vec{I}_1 + \vec{I}_2)^2 = 0, \quad \vec{I}_1 \cdot \vec{I}_2 = \boxed{-\frac{3}{4}}$$