Griffiths

4.21 (a) $$e^{i\pi\sigma_z/2} = \exp\left(i\frac{\pi}{2}\sigma_z\right)$$

$$= \mathbb{1} + \frac{i\pi}{2}\sigma_z - \frac{\left(\frac{\pi}{2}\right)^2}{2!}\mathbb{1} - i\frac{\left(\frac{\pi}{2}\right)^3}{3!}\sigma_z + \frac{\left(\frac{\pi}{2}\right)^4}{4!}\mathbb{1} + \cdots$$

$$= \left(1 - \frac{\left(\frac{\pi}{2}\right)^2}{2!} + \frac{\left(\frac{\pi}{2}\right)^4}{4!} + \cdots\right)\mathbb{1}$$

$$+ i\left(\frac{\pi}{2} - \frac{\left(\frac{\pi}{2}\right)^3}{3!} + \cdots\right)\sigma_z$$

$$= \boxed{\cos\left(\frac{\pi}{2}\right)\mathbb{1} + i\sin\left(\frac{\pi}{2}\right)\sigma_z}$$

$$= i\sigma_z$$

(b) $$U \approx \begin{pmatrix} ? & ? & ? \\ 0 & 1 & 0 \\ ? & ? & ? \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ? \\ y \\ ? \end{pmatrix}.$$

~~A~~ We expect $$U = \begin{pmatrix} \cos\pi & & \sin\pi \\ & 1 & \\ -\sin\pi & & \cos\pi \end{pmatrix}$$

$$= \begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}.$$

$$U\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -x \\ y \\ -z \end{pmatrix}.$$

$$\Rightarrow U\begin{pmatrix} p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} -p_y \\ p_y \\ -p_z \end{pmatrix}.$$

$$\Rightarrow (\hat{r}_x p_y - \hat{r}_y p_z) = (\hat{x} p_y - \hat{y} p_x)$$

will be mapped to $-(\hat{x} p_y - \hat{y} p_x)$.

This clearly flips the signs of angular momentum.

(c). See arxiv 1312.3824 by Steane, exercise 2.