Griffiths

4.17 (a). $\hat{S}_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \frac{i\hbar}{2}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

$$\begin{pmatrix} & -i \\ i & \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -ib \\ ia \end{pmatrix}.$$

impose $\quad -ib = ca$

$\quad ia = cb.$

$$\Rightarrow i\left(\frac{-ib}{c}\right) = cb.$$

$$\frac{b}{c} = cb. \Rightarrow c^2 = 1.$$

$$c = \pm 1,$$

letting $c = 1$, $\quad -ib = a,\quad i(-ib) = b$

$\quad ia = \cancel{c}b,\quad b = b$

From $-ib = ca$, c being real implies one of a, b is real, the other is imaginary,

by guessing, $\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} 1 \\ i \end{pmatrix}$

$$\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -i \end{pmatrix} = \begin{pmatrix} -1 \\ i \end{pmatrix}$$

$\Rightarrow$ the eigenvalues are $\pm\frac{\hbar}{2}$, the eigenvectors are

$$\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}, \quad \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ i \end{pmatrix},$$

(b). $\hat{S}_y \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \frac{\hbar}{2} \begin{pmatrix} & -i \\ i & \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$.

$$= \frac{\hbar}{2} \begin{pmatrix} -i\beta \\ i\alpha \end{pmatrix}$$

We can only expect results of $\pm \frac{\hbar}{2}$.

In general, $\frac{\hbar}{2} \begin{pmatrix} -i\beta \\ i\alpha \end{pmatrix}$ would not be an eigenstate of $\hat{S}_y$, but it can be a superposition of the eigenstates, the components of which will have to depend on α, β.