Goldstein 8.7 Hamilton eqn: $\frac{\partial H}{\partial q_i} = -\dot{p}_i$, $\dot{q}_i = \frac{\partial H}{\partial p_i}$

The two Hami are:

$$H(x,p,t) = \frac{p^2}{2m} + \frac{k}{2}(x - v_0 t)^2$$

$$H(x',p') = \frac{(p' - mv_0)^2}{2m} + \frac{kx'^2}{2} - \frac{mv_0^2}{2}$$

eqm (x,p)

$$\begin{cases} -\dot{p} = \frac{\partial H}{\partial x} = k(x - v_0 t) \\ \dot{x} = \frac{\partial H}{\partial p} = p/m \Rightarrow p = m\dot{x} \end{cases}$$

combined to give $\boxed{-m\ddot{x} = k(x - v_0 t)}$

eqm (x',p'):

$$\begin{cases} -\dot{p}' = \frac{\partial H}{\partial x'} = kx' \\ \dot{x}' = \frac{\partial H}{\partial p'} = (p' - mv_0)/m \Rightarrow p' = m\dot{x}' + mv_0 \end{cases}$$

combined to give $\boxed{-m\ddot{x}' = kx'}$

The equation $-m\ddot{x}' = kx'$ is solved via $x' = \sin(\omega t)$

- - - $-m\ddot{x} = k(x - v_0 t)$ is solved via $x = \sin(\omega t) + v_0 t$,

which agrees with the definition $x' = x - v_0 t$.

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