Goldstein
8.2

$$L \to L' = L + \frac{d}{dt}F(q_1, q_2, \dots, t)$$

$$p_i = \frac{\partial L}{\partial \dot{q}_i} \to p_i' = p_i + \frac{d}{d\dot{q}_i}\frac{dF}{dt}$$

EQN ① $$-\dot{p}_i = \frac{\partial H}{\partial q_i} \to -\dot{p}_i' = \frac{\partial H'}{\partial q_i} = \frac{\partial H}{\partial q_i} - \frac{d}{dq_i}\frac{d}{dt}F$$

$$-\dot{p}_i - \underbrace{\frac{d}{dt}\frac{d}{d\dot{q}_i}\frac{dF}{dt} = \frac{\partial H}{\partial q_i} - \frac{d}{dq_i}\frac{d}{dt}F}$$

equal because it satisfies Lagr eqm.

$$\Rightarrow \boxed{-\dot{p}_i = \frac{\partial H}{\partial q_i}}$$

Eqn ②: $$\dot{q}_i = \frac{\partial H}{\partial p_i} \to \dot{q}_i = \frac{\partial H'}{\partial p_i'}$$

we want to show $\frac{\partial H}{\partial p_i} = \frac{\partial H'}{\partial p_i'}$

Observing the definition $H = \dot{q}_i p_i - L(q_i, \dot{q}_i, t)$

$$H' = \dot{q}_i p_i' - L'(q_i, \dot{q}_i, t)$$

Since L, L' are independent of p_i, p_i', it's clear that

$\frac{\partial H}{\partial p_i} = \frac{\partial H'}{\partial p_i'}$, thus $\dot{q}_i = \frac{\partial H'}{\partial p_i'}$

reduces to $\dot{q}_i = \frac{\partial H}{\partial p_i}$

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