Goldstein 8.1 (a)

The Hami is given by $H = p_i \dot{q}_i - L$, then

$$L = p_i \dot{q}_i - H$$

with p_i defined by $p_i = \frac{\partial L(q, \dot{q}, t)}{\partial \dot{q}_i}$

Then $$\frac{\partial L}{\partial q_i} = -\frac{\partial H}{\partial q_i}$$

The Hamilton eqn $\left(-\dot{p}_i = \frac{\partial H}{\partial q_i},\ \dot{q}_i = \frac{\partial H}{\partial p_i}\right)$ then gives

$$\frac{\partial L}{\partial q_i} = \dot{p}_i$$

$$\Rightarrow \quad \dot{p}_i - \frac{\partial L}{\partial q_i} = 0.$$

$$\boxed{\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = 0}$$

(b) $L' = -\dot{p}_i q_i - H(q, p, t) \Rightarrow \quad \frac{\partial L'}{\partial q_i} = -\dot{p}_i - \frac{\partial H}{\partial q_i} = -2\dot{p}_i$

$$\Rightarrow \boxed{2\frac{d}{dt}\left[\frac{\partial L'}{\partial \dot{q}_i}\right] - \frac{\partial L'}{\partial q_i} = 0}$$

Davidson Cheng
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