Goldstein 8.15 (in progress)

$$L = \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + b q_1^2} + k_1 q_1^2 + k_2 \dot{q}_1 \dot{q}_2$$

$$p_1 = \frac{\partial L}{\partial \dot{q}_1} = 2\dot{q}_1 + k_2 \dot{q}_2, \quad p_2 = \frac{\partial L}{\partial \dot{q}_2} = \frac{2\dot{q}_2}{a + b q_1^2} + k_2 \dot{q}_1$$

Apply transformation $H = p_i \dot{q}_i - L$

$$H = 2\dot{q}_1^2 + k_2 \dot{q}_2 \dot{q}_1 + \frac{2\dot{q}_1 \dot{q}_2}{a + b q_1^2} + k_2 \dot{q}_1 \dot{q}_2 - L$$

$$= \dot{q}_1^2 + \frac{\dot{q}_2^2}{a + b q_1^2} + k_2 \dot{q}_1 \dot{q}_2 - k_1 q_1^2$$

$$p_1^2 = 4\dot{q}_1^2 + k_2^2 \dot{q}_2^2 + 4 k_2 \dot{q}_1 \dot{q}_2$$

$$p_2^2 = \frac{4\dot{q}_2^2}{(a + b q_1^2)^2} + k_2^2 \dot{q}_1^2 + \frac{4 k_2}{a + b q_1^2} \dot{q}_1 \dot{q}_2$$

$$p_1 p_2 = \frac{4}{a + b q_1^2} \dot{q}_1 \dot{q}_2 + 2 k_2 \dot{q}_1^2 + \frac{2 k_2}{a + b q_1^2} \dot{q}_2^2 + k_2^2 \dot{q}_1 \dot{q}_2$$

$$A p_1^2 + B p_2^2 + C p_1 p_2 = \left(4A + B k_2^2 + C k_2\right) \dot{q}_1^2$$

$$+ \left(k_2^2 A + \frac{4B}{(a+bq_1^2)^2} + \frac{2k_2 C}{a+bq_1^2}\right) \dot{q}_2^2$$

$$+ \left(4k_2 A + \frac{4k_2}{a+bq_1^2} B + \left(\frac{4}{a+bq_1^2} + k_2^2\right) C\right) \dot{q}_1 \dot{q}_2$$

Set up system of eq.: to solve for A, B, C:

$$\begin{cases} 4A + B k_2^2 + C k_2 = 1 \\ k_2^2 A + \frac{4}{(a+bq_1^2)^2} B + \frac{2k_2}{a+bq_1^2} C = \frac{1}{a+bq_1^2} \\ 4k_2 A + \frac{4k_2 B}{a+bq_1^2} + \left(\frac{4}{a+bq_1^2} + k_2^2\right) C = k_2 \end{cases}$$