

Jackson 6.4 (b) scratch

$$3x^2 - 1 = 2P_2$$

$$1 = P_0$$

$$\langle P_2 | P_0 \rangle = 0$$

$$Q_{33} = \int (3z^2 - r^2) \rho(\vec{x}') d^3x'$$

$$= \rho \int (3r^2 \cos^2 \theta' - r^2) d^3x'$$

$$= \rho (2\pi) \int r^4 (3\cos^2 \theta' - 1) \sin \theta' d\theta' dr$$

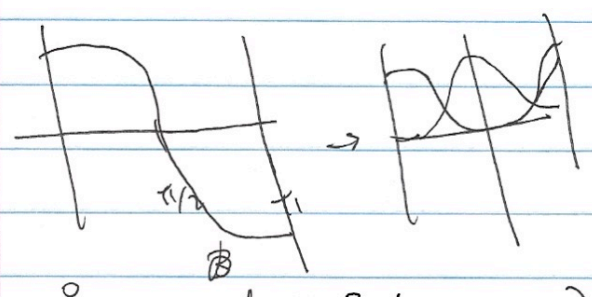
$$= \frac{2\pi\rho}{5} R^5 \int_0^\pi (3\cos^2 \theta' - 1) \sin \theta' d\theta'$$

$$= \frac{2\pi\rho}{5} R^5 \int_{-1}^1 (3x^2 - 1) dx = \frac{2\pi\rho}{5} R^5 [x^3 - x]_{-1}^1$$

$$= \frac{4\pi}{5} \rho R^5$$

Recall $\rho = \frac{-m\omega}{\pi c^2 R^3} \Rightarrow Q_{33} = -\frac{m\omega}{\pi c^2 R^3} \frac{4\pi}{5} R^5$

$$= -\frac{4m\omega R^2}{5c^2}$$



$$\frac{d}{d\theta} (3\cos^3 \theta' + \cos \theta') = 3\cos^2 \theta' (-\sin \theta') - \sin \theta'$$

$$\frac{d}{d\theta'} = -3$$

$$\frac{d}{d\theta} [-\cos^3 \theta' + \cos \theta']$$

$$= -3\cos^2 \theta' (-\sin \theta') - \sin \theta'$$

$$= [3\cos^2 \theta' - 1] (\sin \theta')$$

$$[-\cos^3 \theta' + \cos \theta']_0^\pi$$

$$= [-(-1)^3 + (-1)] = 1 - 1$$

$$-[-1 + 1]$$

(b)
G4 scratch

$$Q_{33} = \int (3z^2 - r^2) \rho(\vec{r}') d^3x'$$

$$= \rho \int (3r'^2 \cos^2 \theta' - r'^2) d^3r'$$

$$= \rho \int r'^2 [3 \cos^2 \theta' - 1] d^3r'$$

$$= \rho(2\pi) \int r'^4 [3 \cos^2 \theta' - 1] \sin \theta' dr' d\theta'$$

$$= \rho(2\pi) \frac{1}{5} R^5 \int_0^\pi (3 \cos^2 \theta' - 1) \sin \theta' d\theta'$$

$$= \rho \int_{-1}^1 [3(1-x^2) - 1] dx$$

$$= \int_{-1}^1 [3(1-x^2) - 1] dx = \frac{2\pi\rho}{5} R^5 \int_{-1}^1 (3x^2 - 1) dx$$

$$= \int_{-1}^1 [3x^2 + 2] dx$$

$$= \frac{2\pi\rho}{5} R^5 [x^3 - x]_{-1}^1$$

$$(1-1) - (-1 - (-1))$$

$$= [-x^3 + 2]_{-1}^1$$

$$= \frac{2\pi\rho}{5} R^5 [(1-1) - (-1-1)]$$

$$= 0 - (-2)$$

$$= [-1 - (-1)] = -2$$

$$= \frac{2\pi\rho}{5} R^5 [0 - (-2)]$$

$$= \frac{4\pi\rho}{5} R^5$$

$$\frac{-mw}{c^2 R^3} \frac{4\pi R^5}{5} = \frac{-mw 4}{c^2 R^3}$$

$$\frac{4\pi\rho R^5}{5} = \frac{4\pi}{5} \frac{-mw}{c^2 R^3} R^5$$

$$= \frac{4}{5} \frac{mw R^2}{c^2} \frac{4}{5}$$

$$= \frac{4}{5} \frac{mw R^2}{c^2}$$