Jackson 6.2 (a)

$$\delta[\vec{x}' - \vec{r}(t_{ret})] = \delta(f(\vec{x}')), \quad f(\vec{x}') = \vec{x}' - \vec{r}(t_{ret})$$

$$\frac{\partial f}{\partial \vec{x}'} = 1 - \frac{\partial}{\partial \vec{x}'} \vec{r}(t_{ret})$$

$$= 1 - \frac{\partial \vec{r}(t_{ret})}{\partial t_{ret}} \frac{\partial t_{ret}}{\partial \vec{x}'}$$

$$= 1 - \vec{v} \cdot \frac{\partial t_{ret}}{\partial \vec{x}'}$$

$$t_{ret}(\vec{x}') = t - \frac{|\vec{x} - \vec{x}'|}{c}$$

$$= t - \frac{1}{c}\sqrt{(x_1 - x_1')^2 + (x_2 - x_2')^2 + (x_3 - x_3')^2}$$

$$\frac{\partial t_{ret}}{\partial x_1'} = -\frac{1}{c}\frac{1}{2}\frac{1}{\sqrt{(x_1 - x_1')^2 + (x_2 - x_2')^2 + (x_3 - x_3')^2}} 2(x_1 - x_1')(-1)$$

$$= \frac{1}{c}\frac{x_1 - x_1'}{|\vec{x} - \vec{x}'|} = \frac{1}{c}\hat{R}_1$$

$$\Rightarrow \frac{\partial t_{ret}}{\partial \vec{x}'} = \frac{1}{c}\hat{R}, \quad \frac{\partial f}{\partial \vec{x}'} = 1 - \frac{\vec{v} \cdot \hat{R}}{c} = K$$

$$\Rightarrow \int d^3x' \, \delta[\vec{x}' - \vec{r}(t_{ret})] = \int d^3\vec{x}' \frac{1}{K} \delta(\vec{x}' - \vec{x}^*), \quad \text{where } \vec{x}^*$$

is the root of $f(\vec{x}')$, that is, $\vec{x}^* - \vec{r}(t_{ret}) = 0$.

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